6

Lenses, Apertures, and Resolution

CHAPTER PREVIEW

Electron lenses are the TEM's equivalent of the glass lenses in a visible light microscope (VLM) and, to a large extent, we can draw comparisons between the two. For example, the behavior of all the lenses in a standard TEM can be approximated to the action of a convex (converging) glass lens on monochromatic light. The lens is basically used to do two things

- Take all the rays emanating from a point in an object and recreate a point in an image
- Focus parallel rays to a point in the focal plane of the lens

The lens can't collect *all* the rays from the object and we often deliberately limit the collection angle with an aperture. We can draw ray diagrams showing how electron lenses control beams of electrons. These diagrams correspond directly to the ray diagrams used in physical optics. Of course the analogy with light fails for certain aspects, but basically it will pervade this chapter. So we'll start by reminding you of the principles of light optics insofar as they relate to electron optics. Then we'll discuss the electron lens in more detail, showing how an electron behaves as it passes through such a lens. We'll describe some actual lenses and tell you how we use different kinds of electron lenses to do different things in the microscope.

A major limit to the use of electron lenses is the fact that we aren't very good at making them. They suffer from rather severe spherical and chromatic aberrations which we usually control by inserting limiting apertures to select electrons nearest to the optic axis since these are least affected by the lens aberrations. Recent technical developments have permitted these aberrations to be largely overcome, but aberration-corrected TEMs are both rare and expensive; most microscopists still have to live with these limitations. So you need to understand lens aberrations, since they play a major role in deciding what we can and cannot do with the microscope. In particular, lens aberrations (rather than the wavelength of the electrons) limit the resolution of the TEM (unlike in the VLM). Since resolution is often the single most important reason for buying a TEM, you need a firm understanding of this concept. Unfortunately, we electron microscopists aren't always very precise in our definitions of resolution. Finally, we describe how the apertures we use aid both the depth of field and the depth of focus of the instrument.

6.1 WHY LEARN ABOUT LENSES?

Why should we learn about electron lenses? As in a VLM, the lenses in a TEM control all the basic operational functions of the instrument. As you are well aware, we have to physically move glass lenses up and down in a VLM to control the intensity of the illumination and to focus the image. The focal length of a glass lens is fixed so we have to change lenses to change the magnification. We choose stronger lenses for higher magnification. By contrast, in a TEM, the positions of the lenses are fixed but we can change the strength of the lens at will.

CHANGING THE LENS

We change focus, change the intensity of illumination or change magnification by changing the strength of the lenses.

As you'll see, in most cases the lenses we use are electromagnetic, so we change their strength by

changing the current through a coil around a soft-iron core which changes the strength of the resultant magnetic field. Almost any operation we carry out on the TEM involves changing magnification or focus; we use electron lenses to magnify and focus the electron beam, the images, and the DPs.

These factors are critical in the principal functions of a TEM: imaging, diffraction, and analysis which, respectively, comprise the next three parts of this book. An aperture is used to select different electron beams to form different images, thus manipulating the image contrast. Another aperture is used to select different regions of the specimen to contribute to the DP as we'll see in Chapter 9.

So knowing how these aperture/lens combinations work allows you to understand how we control the TEM and why we do certain operations on the microscope.

APERTURES

We use apertures in the lenses to control the divergence or convergence of electron paths through the lenses which, in turn, affects the lens aberrations and controls the current in the beam hitting the specimen.

An understanding of electron lenses will help us to answer such questions as

- Why can we see finer detail with an electron microscope than with a light microscope?
- Why can't we see as much detail as we might expect from physics?
- Why does the TEM have a better depth of field and depth of focus than the VLM?

We'll see that the answers to these questions lie in the quality of the lenses, and how we use them. In this chapter we'll discuss the basics of how a lens/aperture combination works. Throughout the book you'll come across different uses and combinations of lenses and apertures. So this is a central chapter for the serious microscope operator but it is only an introduction to the important aspects of electron optics, which is a field in itself. For this you need to explore Chapter 2 in the companion text and the electron-optics texts in the references. Apart from the tremendous advances brought about by aberration correction, electron optics is relatively static these days but traditional light optics is undergoing a renaissance from which electron optics can only benefit. If you are interested you should consult any of the optics textbooks that we referenced back in Chapter 2 and check out URL #1.

6.2 LIGHT OPTICS AND ELECTRON OPTICS

You are already familiar with the action of a magnifying glass lens on light rays. The magnifying glass is a convex lens. It can be used in two ways to control the light rays coming through it. First, it can produce a magnified image of the object you're looking at. Second, it can focus a parallel beam of light to a point, in the focal plane of the lens. (When younger, we've all used this latter property to set something or someone on fire by focusing the (parallel) sun's rays.) These two actions, forming an image of an object and focusing parallel rays to a point, are all we need in order to understand how the lenses in a TEM work. The reason that we can get away with this simple approach is because the electron lenses act, to a reasonable approximation, like convex glass lenses; in detail they're often equivalent to more complex combinations of convex lenses and aberration correction involves the equivalent of a divergent or concave lens. We introduce the practical use of lenses in the TEM in Chapter 9.

6.2.A How to Draw a Ray Diagram

In traditional light optics it's customary to draw diagrams of the paths of light rays through the lens and these ray diagrams are usually drawn horizontally because the traditional optical bench on which light-optical experiments are carried out is a horizontal setup. Likewise, we draw diagrams of the electron trajectories through electron lenses but, since the TEM is a vertical instrument, we will draw all our ray diagrams vertically assuming the gun is at the top of the column of lenses (although this isn't invariably so, as we describe in Chapter 9).

Let's start by drawing ray diagrams to illustrate the two fundamental lens actions of image formation and the focusing of parallel rays. In these and all subsequent diagrams we'll draw all the lenses in the TEM as convex lenses. We will draw all electron ray paths as straight lines outside the lens, and we'll start by assuming that the lenses are perfect. We'll also draw the lenses as so-called 'thin' lenses, which means their thickness is small compared to their radii of curvature. Actually, we'll make the lenses *very* thin. We'll see that these assumptions are all precisely wrong, yet sufficiently reasonable that traditional ray diagrams are nonetheless very useful.

The first thing we need to do is to have a base line on which to draw our diagrams; this line is called the optic axis (also called the rotation axis in the TEM because, as you'll learn, the electrons actually rotate through the lens even though we draw the ray paths as straight lines).

THE OPTIC AXIS

An imaginary line down the column of the TEM passing through the center of each lens.

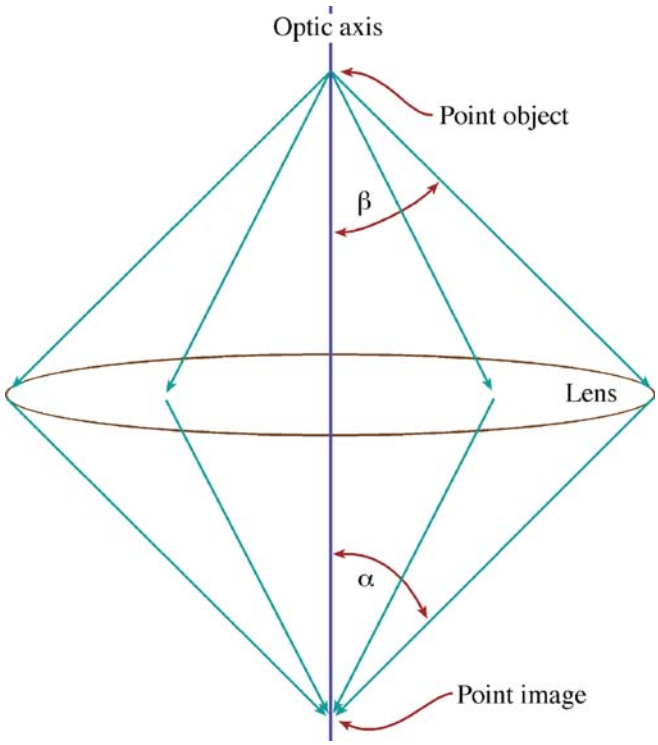

FIGURE 6.1. Image formation by a convex lens. A point object is imaged as a point and the collection angle of the lens is defined relative to the object (β) or the image (α).

Now the first action of a lens that we want to show is how it produces an image of an object. In a TEM the object will usually be the specimen itself or an image of it, but it may also be the electron source, which is an object for the illumination system. If we assume the object is a point and the radiation is emanating from that point (a so-called 'self-luminous object'), then a perfect lens will gather a fraction of that radiation and form a point image. This action is shown in Figure 6.1 in which the point is on the optic axis. The fraction of the rays from the object gathered by the lens is an important variable, defined by the angle β in Figure 6.1. Ultimately, as you can see, β is governed by the size of the lens, but we often choose to limit β by inserting an aperture, as we'll discuss later in this chapter. You'll often see the angle of collection defined as α, but we will reserve α for convergence angles (see Section 2.7). From now on, as we did in earlier chapters, we'll talk about angles when we actually mean semi-angles.

LENSES ARE FINITE

All lenses are imperfect insofar as they cannot gather all the radiation emitted by an object and so can never create a perfect image.

However, as you know from Chapters 2 to 4, most electrons are strongly forward scattered, so we can in practice gather a high fraction of the scattered electrons.

The angles in Figure 6.1 and in the other ray diagrams we'll draw are all greatly exaggerated.

EXAGGERATE ANGLES

In practice, a typical value of β is maybe a few tens of milliradians (10 mrad ~ 0.57°) so if the diagrams were drawn to scale they would be many times longer than they were wide and all the ray paths would be exceedingly narrow. Since drawing to scale is impractical, we always exaggerate the angles considerably in all electron ray diagrams.

If the object has a finite size, we can illustrate this by an arrow, asymmetrically positioned with respect to the optic axis, as in Figure 6.2. Then the lens creates an image of the arrow, rotated by 180°. To draw this figure, the first step is to draw line 1 from the arrowhead through the center of the lens, because rays crossing the optic axis in the lens (or on-axis rays which travel down the axis) are *not* affected by the lens at all and remain as a straight line.

The second step is to draw line 2 which is a ray from the arrowhead that is parallel to the optic axis. We could draw such a ray from any point along the arrow and the further away rays are from the optic axis, the more strongly they are bent by a convex lens. So we take line

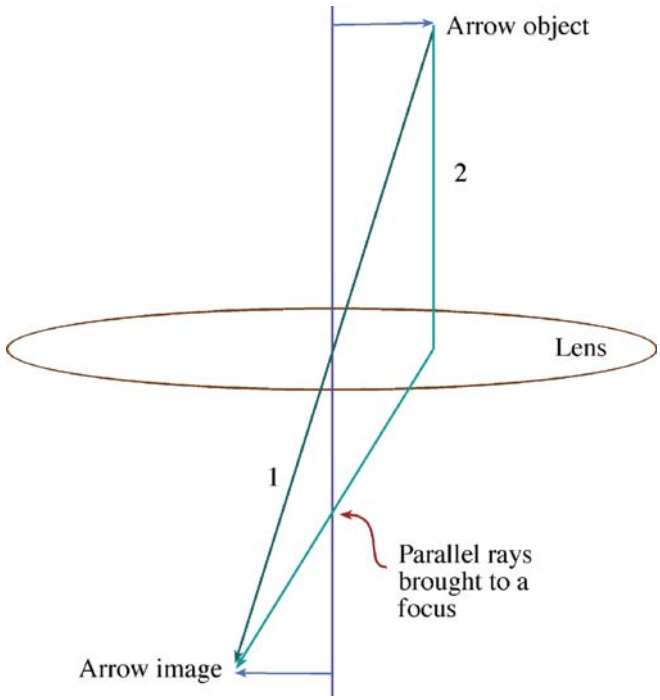

FIGURE 6.2. How to draw a ray diagram: first construct ray path #1 through the middle of the lens, then draw ray path #2 (initially parallel to the optic axis) to determine the lens strength. Where path #2 intersects the optic axis defines the focal plane.

2 and bend it toward the optic axis as it passes through the lens. We can choose to make the lens as strong as we wish, and the strength determines how much the ray is bent and where lines 1 and 2 meet to recreate an image of the arrowhead. Where ray path 2 intersects the optic axis (and thus intersects a ray path along the axis) defines the focal plane of the lens and thus illustrates the second fundamental action of a convex lens, i.e., the lens brings rays that are initially parallel to a focus.

FOR A THIN LENS

A fundamental principle of how a lens works is that an electron passing through the middle of the lens is unaffected so we can draw its path as a straight line. All other electron paths are bent when they pass through the lens.

Some important points on electromagnetic lenses

- The strength of the lens determines where the parallel electrons are focused: stronger lenses having shorter focal lengths.
- The focal plane is where initially parallel rays intersect after passing through the lens.
- The image formed by the lens is rotated by 180° with respect to the object.

Now a full ray diagram for an object of finite size, symmetrically positioned about the axis, combines aspects of Figures 6.1 and 6.2, as shown in Figure 6.3. In Figure 6.3, all rays from a point in the object are brought back to a point in the image and all parallel rays (whether parallel to the optic axis or not) are brought to a focus in a plane at a position depending on their angle to the axis.

Note that on-axis parallel rays are focused on axis and off-axis parallel rays are focused off axis.

This is a most important property, since it allows the lens to create DPs in the focal plane. We'll use this diagram to introduce you to the principal terms used in lens optics.

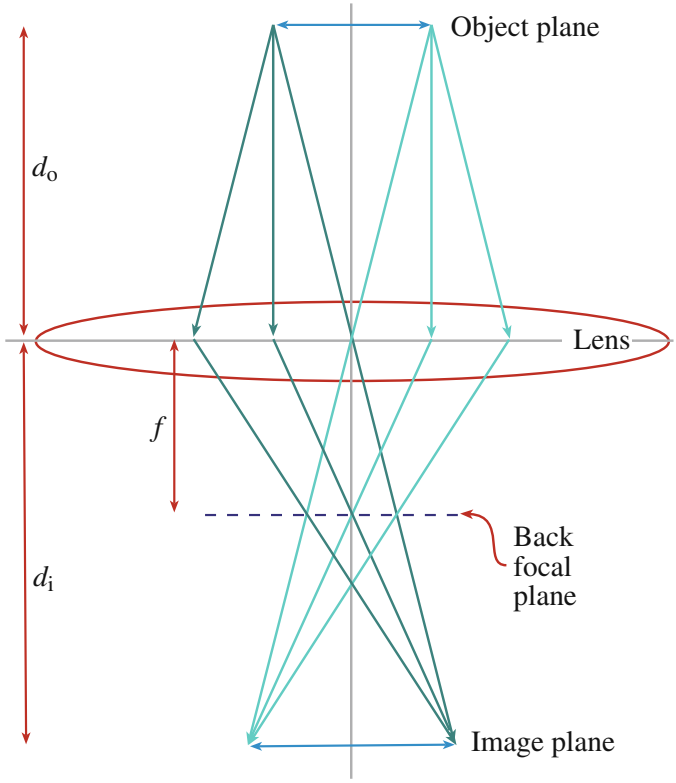

FIGURE 6.3. A complete ray diagram for a finite object, symmetrically positioned around the optic axis. All rays emerging from a point in the object (distance d_o from the lens) that are gathered by the lens converge to a point in the image (distance d_i from the lens) and all parallel rays coming from the object are focused in the focal plane (distance f from the lens).

6.2.B The Principal Optical Elements

From the above diagrams, we can define several *principal planes* to which we will often refer. The first plane is the plane of the lens. In a thin lens this plane can be imagined as a line through the middle of the lens. The object plane is the plane containing the object point in Figure 6.1 or the object arrow in Figures 6.2 and 6.3. The object plane always lies above the lens in question in the diagrams in this text. The image plane (sometimes called the Gaussian-image plane) is the plane containing the image point or arrow and it always lies below the lens. These two planes are said to be conjugate, which means optically equivalent. Rays leaving a point in one plane are brought to a point (if the lens is perfect) in a conjugate plane and vice versa. In other words, the electron doesn't care which way it goes through the lens and this is the basis for the theorem of reciprocity which we'll discuss when we compare TEM and STEM imaging in Chapter 9. The focal plane of the lens is the plane in which the parallel rays are brought to a focus as shown in Figures 6.2 and 6.3. In the image-forming process in a TEM, the focal plane lies after or 'behind' the lens and so the plane is sometimes called the back-focal plane (BFP). There is also an equivalent front-focal plane (FFP) and a convex lens would take all the rays coming from a point in the front-focal plane and create a parallel beam of radiation, in exactly the reverse manner to Figures 6.2 and 6.3.

6.2.C The Lens Equation

From the above diagrams we can define three important distances, labeled in Figure 6.3: the distance from the object plane to the lens (the object distance d_o), the distance from the lens to the image plane (the image distance d_i), and the distance from the lens to the back-focal plane (the focal

length f). Now if the lens is symmetric in strength either side of the lens plane (i.e., the front and back-focal planes are the same distance from the lens) then we can write the following basic equation

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} \tag{6.1}$$

THE PLANES
The principal planes of a lens comprise the object, image and focal planes.

which is known as Newton's lens equation. You'll find a proof in any standard optics text (several were referenced back in Chapter 2). The distances d_o and d_i are measured from the two different principal planes in a thick lens, but from the same plane in the middle of a thin lens, which we are assuming here. In all cases that we'll consider, the object distance (and therefore the image distance) is greater than the focal length. Thus a real image is produced on the other side of the lens beyond the back-focal plane. If the object were within the (front) focal length, then a virtual image would be produced on the same side of the lens as the object, and this is often the case in light optics. Since we don't deal with virtual images in the TEM we'll ignore this aspect.

6.2.D Magnification, Demagnification, and Focus

We can use Newton's lens equation to define the magnification of a convex lens as

$$M = \frac{d_i}{d_o} \tag{6.2}$$

M is also approximately equal to the ratio of the collection angles of the lens subtended at the object (β) and at the image (α) as shown in Figure 6.1, assuming that these angles are small, as they invariably are in a TEM. In this example the magnification is unity.

STRENGTH VERSUS MAGNIFICATION
Under conditions normally found in the TEM, strong lenses *magnify less* and *demagnify more*. In VLMs stronger lenses produce greater magnifications.

Now we may sometimes want to *demagnify* an object (for example, when we want to form a small image of the electron source, to create the smallest possible probe at the specimen). If that is the case, we define the demagnification as $1/M$. In a VLM we could change the magnification by moving the object relative to the lens or vice versa, and adjusting our eyes accordingly, but generally we rotate in another objective lens of different strength (curvature). In a TEM we change magnification in this latter way by changing the strength of the lens, but you'll see that we can do this without changing the lens itself. So electron lenses differ fundamentally from glass lenses in that one lens can be adjusted to a range of strengths.

If we make the lens stronger, then the focal length is shortened as shown in Figure 6.4. If f is shortened but d_o is unchanged, then d_i must be correspondingly shorter and the image magnification is smaller, or the demagnification is larger.

How do we get the high magnifications that we need to form images of atomic columns such as Figure 1.2? Since, as you'll see in Chapter 9, we tend to operate the objective lens of the TEM at a fixed strength, we move the object plane close to the lens thus making d_o small and M correspondingly large (see equation 2). We then make the image plane of the first lens, the object plane for the next lens and repeat this for several lenses in tandem one after the other. So we end up with a multi-lens system like a compound VLM. We'll discuss many more details of lens combinations in the illumination and imaging systems of the TEM in Chapter 9.

Now, in principle, there's nothing to stop us magnifying as much as we wish. However, above a certain magnification, we will see no more information because

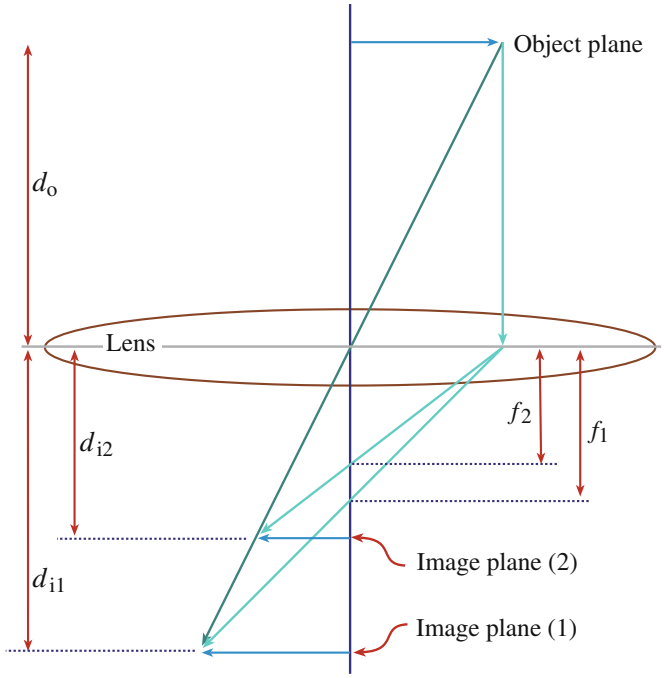

FIGURE 6.4. Strengthening the lens shortens the focal length f. So a weaker lens ($f1$) produces a higher magnification of the object than a stronger lens ($f2$) since the image distance d_i increases, but the object distance, d_o, is unchanged.

other factors limit the image detail and therefore the resolution of the microscope. We'll discuss this point later in Section 6.6. We'll also see that there are times when we want to look at an image of the focal plane (because this contains the DP). To do this, the back-focal plane of the upper lens must become the object plane for the subsequent lenses in the imaging system.

MAGNIFICATION VERSUS RESOLUTION
Don't confuse the two.

When discussing the focus of images we need another convention because we'll find that there is much useful information to be gained and certain technical advantages to operating out of focus. This situation is somewhat different to almost any other form of microscopy wherein out-of-focus images are generally less useful or, more likely, completely useless. However, in TEM we need to define the following two conditions relative to the plane in which a focused image is formed

- If the lens strength is increased such that the image forms above (i.e., before the rays get to) the image plane, then the image will be out of focus and we say the lens is *overfocused*.
- If the lens is weakened and the image forms below (i.e., after) the image plane, the image will be out of focus and the lens is said to be *underfocused*.

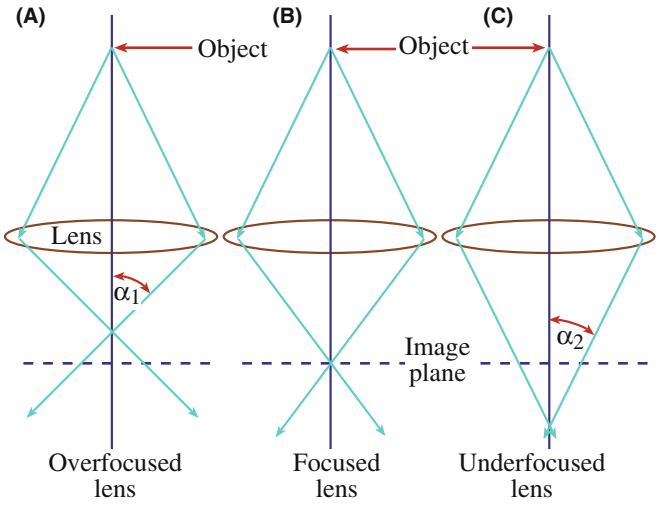

FIGURE 6.5. (A) The concept of overfocus in which a strong lens focuses the rays from a point in the object above the normal image plane where a focused image (B) of the object is usually formed. At underfocus (C) the lens is weakened and focuses the rays below the image plane. It is clear from (C) that at a given underfocus the convergent rays are more parallel than the equivalent divergent rays at overfocus ($\alpha_2 < \alpha_1$).

It's very easy to confuse these two terms unless you think in terms of the vertical frame of the microscope as shown in Figure 6.5. One point to note from Figure 6.5, which we'll find useful, is that the electrons are closer to being parallel to the optic axis when the lens is underfocused than when it is overfocused.

WEAK LENS
A weak, underfocused lens gives a more parallel electron beam. Remember α_1 and α_2 are very small.

We'll exploit underfocused imaging conditions on many occasions in the future. We'll also find there are times when we should operate with our DPs out of focus and also get different information to when it is in focus. So even dexterously challenged TEM operators or those with aging eyes can still do well!

6.3 ELECTRON LENSES

Electrons were first successfully focused by Busch in 1927; he used an electromagnet of the sort that Ruska later incorporated into the first TEM shown in Figure 1.1. Busch also showed that it was possible to focus electrons using electrostatic fields and we've already seen how this works in thermionic electron guns in Chapter 5. In practice, magnetic lenses are superior in many respects, particularly because they are not susceptible to high-voltage breakdown. The TEMs that we're discussing in this text all use magnetic lenses, so we won't discuss electrostatic lenses further here but they are examined in the companion text.

6.3.A Polepieces and Coils

To make a magnetic electron lens we need two parts. Both are drawn schematically in cross section in Figure 6.6. First there is a cylindrically symmetrical core of soft magnetic material such as soft iron, with a hole drilled through it. We call this soft iron a *polepiece* and the hole is called the *bore* of the polepiece. (Soft refers to the magnetic not the mechanical behavior.) In most lenses there are two polepieces (upper and lower), which can be part of the same piece of soft iron as in Figure 6.6 or they may be two separate pieces. The distance between the polepiece faces is called the *gap* and the bore-to-gap ratio is another important characteristic of such lenses, controlling the focusing action of the lens. Some polepieces are machined to a cone shape; the cone angle is then an important variable in the lens performance.

The second part of the lens is a coil of copper wire which surrounds each polepiece. When we pass a current through the coil, a magnetic field is created in the bore.

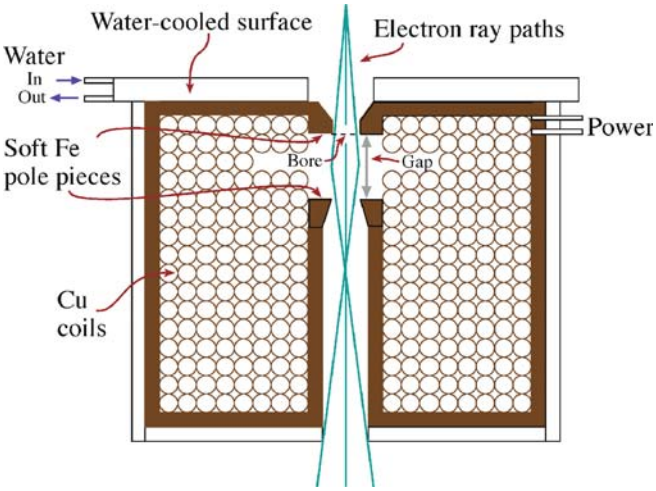

FIGURE 6.6. Schematic diagram of a magnetic lens. The soft-iron polepieces sit in the hole down the middle of the lens and are surrounded by the copper coils through which the current runs to magnetize the polepieces. When viewed in cross section, the bore and the gap between the polepieces are visible. The magnetic field is weakest on axis and increases in strength toward the sides of the polepiece, so the more the electrons travel off axis the more strongly they are deflected.

FIGURE 6.7. A real lens: the cylindrical shape conceals the copper wire coils. The two conical polepieces beside the lens sit inside the central hole in the lens. The three-pin electrical connections provide current to the coil to magnetize the polepieces, and cooling water is circulated in and out of the two holes in the top plate of the lens to dissipate the resistive heat generated in the coils. Compare this picture with the schematic in Figure 6.6.

This field is inhomogeneous along the length of the lens, but axially symmetric. It is the strength of the field in a magnetic lens that controls the electron trajectories or ray paths. As you can see, the electron path through the lens is a reasonable approximation to the schematic diagram back in Figure 6.1.

The resistive heating of the coil means that the lenses have to be cooled and a water recirculating system is an essential part of TEM lenses. A real lens removed from the column of a TEM is shown in Figure 6.7.

6.3.B Different Kinds of Lenses

The principles that we've just described are incorporated into different kinds of lenses used in the TEM. Most lenses in the microscope are weak lenses with large gaps. Either they act to demagnify the source image onto the specimen or they magnify the image or DP from the specimen and project it onto the viewing screen or CCD in ways that we'll see in Chapter 9. Typically these lenses are of the sort shown schematically in Figure 6.6. An aperture can be introduced into the bore of the lens, as we'll discuss later.

PRACTICAL HINT

You should be able to get a readout (on the TEM computer display) of the current through any lens coil. It is a useful thing to know the standard lens currents for your common operating modes such as imaging and diffraction and for creating various beam sizes.

Compared to the other lenses in a TEM, the objective lens is a very strong lens. Several types exist, depending on the needs of the particular TEM. The most flexible objective lens is that in which the upper and lower polepieces are separated and have their own coils as shown in Figure 6.8A. This geometry gives the space needed to allow us to insert both the specimen and the aperture between the polepieces. With this type of polepiece, other instruments such as X-ray spectrometers can have relatively easy access to the specimen. For the same reason, it is straightforward to design specimen holders that do a variety of tasks such as tilting, rotating, heating, cooling, straining, etc. This versatility accounts for the popularity of the split-polepiece lens in TEMs.

With split polepieces it is possible from to make the upper polepiece behave differently from the lower polepiece. The most common application of this is to excite the upper-objective polepiece very strongly. This kind of (asymmetrical) lens is ideal for an AEM/STEM because it can produce both the necessary broad beam of electrons for TEM and a fine beam of electrons for AEM and STEM. We'll see how this is accomplished in more detail in Chapter 9.

If high resolution is a major requirement, then we'll see that it is essential to keep the focal length of the objective lens short and this means a very strong lens is

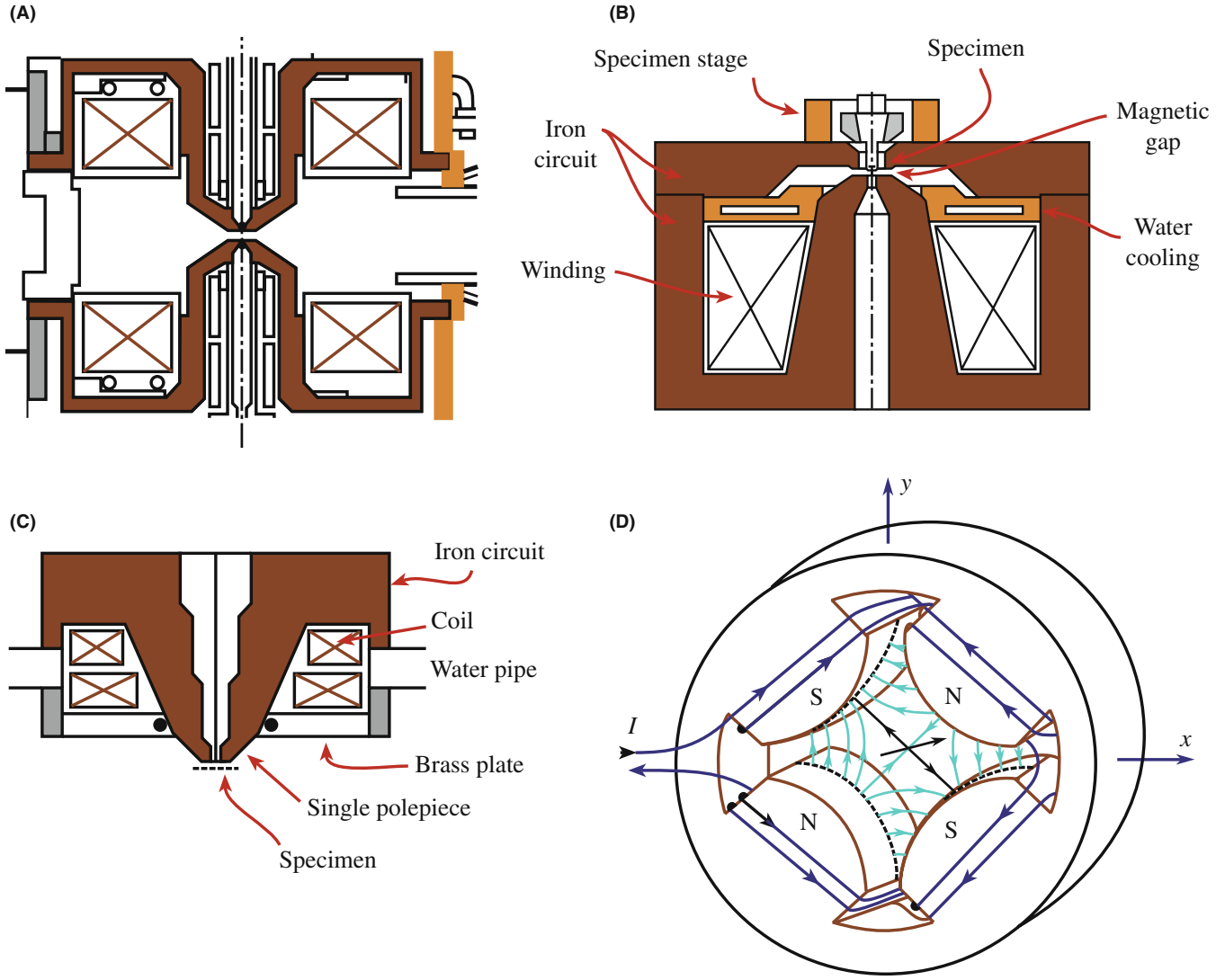

FIGURE 6.8. A selection of different lenses; (A) a split polepiece objective lens, (B) a top-entry immersion lens, (C) a snorkel lens, and (D) a quadrupole lens.

needed. This is traditionally accomplished by using an immersion lens. The specimen is dropped into (i.e., immersed in) the center of the lens field as shown in Figure 6.8B. In such a top-entry stage the specimen is surrounded by the objective lens and so it is a more difficult engineering feat to manipulate, heat or cool the specimen and it is not possible to get X-ray detectors near the specimen, so analytical microscopy is very inefficient. If the focal length is kept really short to give the highest resolution, then it becomes difficult to tilt the specimen more then a few degrees. So in the highest-resolution TEMs you can't do much apart from imaging and diffraction over a restricted range of tilt (see Chapter 8 on stages). This limitation can be overcome by designs such as the snorkel lens as shown in Figure 6.8C, which is a single polepiece lens with a small bore to give a strong lens. Spherical-aberration correction also reduces the need to have the strong lenses for high resolution, so larger gaps are feasible in aberration-corrected TEMs without compromising resolution.

THE OBJECTIVE LENS

The most important lens in the TEM. It forms the images and DPs that are magnified by the other lenses. It is also the most difficult to construct since the specimen must be located close to the plane of this lens.

The limitations of ferromagnetic polepieces can be overcome using superconducting lenses. We cannot make soft-iron polepieces stronger than their saturation magnetization and this limits the focal length and the probe-forming capability of the lens. Superconducting lenses can overcome these limitations but

since a superconductor generates a fixed field, it cannot be varied in the same way as a conventional ferromagnetic lens and so it is not very flexible. Periodically there are increased fluxes of papers describing superconducting lenses because they are small, they don't need water cooling, and they cool the area around the specimen which improves the vacuum, helps minimize contamination, and preserves biological or polymeric specimens. Such lenses also saw a brief flurry of activity after the discovery of high-Tc superconductors. These lenses can generate intense fields (>100 T compared to the maximum of ~2 T in electromagnetic lenses) which are very promising for forming fine probes with high-energy electrons (useful in AEM). Superconducting lenses are so strong that their aberrations (which we'll get to in Section 6.5) are inherently small and they could feasibly be used to construct very compact TEMs.

In addition to these variations on the theme of a single or double polepiece, it is also possible to design a quadrupole, sextupole, or octupole lens in which the focusing action is achieved by four, six, or eight polepieces, respectively. Adjacent polepieces are of opposite polarity as shown in Figure 6.8D. These lenses are not used in TEMs as magnifying lenses but are used to correct lens defects such as astigmatism (see Chapter 9), are used as lenses in aberration correctors (see Section 6.5.A) and also in electron energy-loss spectrometers (Chapter 37). These lenses require less power, and they don't introduce any rotation into the image, which as we'll now show, is a characteristic of standard, electromagnetic lenses.

6.3.C Electron Ray Paths Through Magnetic Fields

We need a bit of mathematics to explain how magnetic lenses actually work. When an electron with charge q (= –e) enters a magnetic field with a strength **B** (Tesla) and an electric field of strength **E**, it experiences a force **F**, known as the Lorentz force, which depends on the velocity of the electron, **v**. All these factors are related through the equation

$$\mathbf{F} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B}) = -\mathrm{e}(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \qquad (6.3)$$

where the term in parentheses is a vector cross-product. Since we are not applying an electric field within the lens, the resulting (Lorentz) force **F** is a vector normal to **v** and **B,** which are inclined to one another at an angle θ. You can easily work out the relative directions of **E**, **v**, **B**, and **F** using the right-hand rule in which your thumb represents the direction of the force acting on a *positive* charge moving in the direction of the middle finger through a field in the direction of the index finger. So the force on the electron acts in the *opposite* direction to your thumb.

RIGHT-HAND RULE
Field: Forefinger
Velocity (Speed): Second finger
Thrust: Thumb

The force on an electron entering a uniform magnetic field, nearly 90° to **B** is

$$F = \mathrm{e}vB\sin\theta = \mathrm{e}vB = \frac{mv^2}{r} \qquad (6.4)$$

where r is the radial distance of the electron from the optic axis (sometimes called the cyclotron radius for historical reasons which you should be able to recognize) and m is the mass of the electron. We can rearrange equation 6.4 to give an expression for

$$r = \frac{mv}{\mathrm{e}B} \qquad (6.5)$$

Since v is a relativistic velocity, we should write this equation as

$$r = \frac{\left[2\mathrm{m}_0 E\left(1 + \frac{E}{2E_0}\right)\right]^{1/2}}{\mathrm{e}B} \qquad (6.6\,\mathrm{A})$$

where m_0 and E_0 are the rest mass and energy of the electron, respectively. This form of the equation allows us to substitute known constants to estimate r (in meters)

$$r = \frac{3.37 \times 10^{-6}\left[V\left(1 + 0.9788 \times 10^{-6}V\right)\right]^{1/2}}{B} \qquad (6.6\,\mathrm{B})$$

In deriving equation 6.4, we made a rather gross oversimplification. If θ equals 90°, the electron is traveling straight down the optic axis and is not focused; in fact it doesn't even notice that a lens is there! It is the *deviation* from θ = 90° that gives the lens effect. The next step, therefore, is to separate the electron velocity **v** in a magnetic field into two components, $\mathbf{v}_1$ perpendicular to, and $\mathbf{v}_2$ parallel to the magnetic-field direction **B**, as shown in Figure 6.9, where $v_1 = v\sin\theta$ and $v_2 = v\cos\theta$. The parallel component, $\mathbf{v}_2$, results in motion parallel to the optic axis in the z direction, with $z = v_2 t$, while the perpendicular component produces circular motion with a radius given by equation 6.5.

THE FIELD
For V = 100 kV and B = 1 Tesla, from equation 6.5 the radius, r, is < 1 mm.

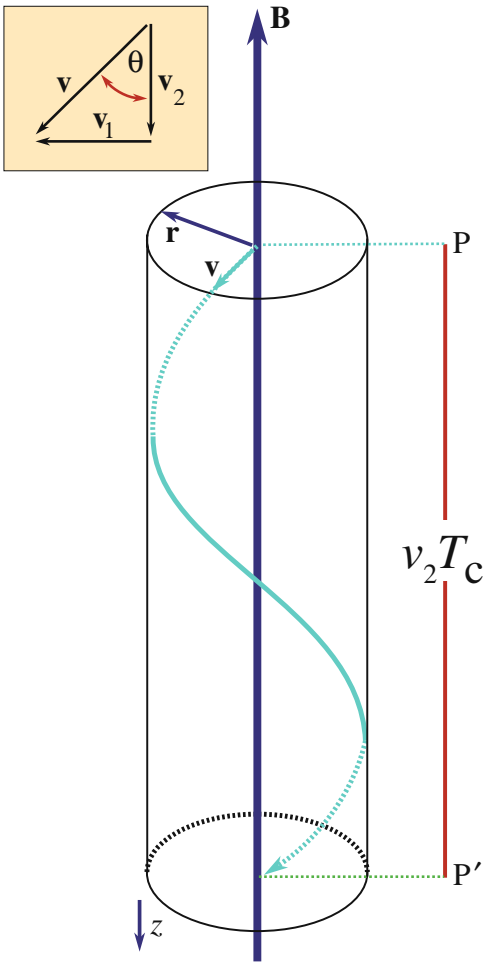

FIGURE 6.9. Electron trajectories in a homogeneous magnetic field, strength **B**. The electrons have velocity components parallel and perpendicular to the field, so long as they are not traveling at 90° to the direction of **B**. The Lorentz force causes electrons passing through point P on the optic axis to spiral through the field and intersect the axis again at P′. The electron's helical path defines the cyclotron radius, *r*.

So all the ray diagrams that we draw ignore this complicating factor which also explains why the optic axis is sometime referred to as the rotation axis. The period of rotation (T_c) through the field gives rise to the (cyclotron) frequency ω_c

$$\omega_c = \frac{2\pi}{T_c} = \frac{eB}{m} \tag{6.7}$$

From these various relationships, we can calculate the complete ray paths through the lens. The most important equations are called the *paraxial* (i.e., near-axis) ray equations. These equations determine both *r* and the angle of rotation (θ) about the axis as the electron moves around the axis in the direction *z*: it rotates under the influence of the rotationally symmetrical field, *B*. These equations, which neglect electron trajectories far off axis, are derived in texts on electron optics. As Hawkes succinctly states "a straightforward, but quite lengthy calculation yields"

$$\frac{d^2r}{dz^2} + \frac{\eta^2 B^2 r}{2\,V^{1/2}} = 0 \tag{6.8}$$

$$\frac{d\theta}{dz} = \frac{\eta B}{2\,V^{1/2}} \tag{6.9}$$

where V is the accelerating voltage of the microscope and η is $(e/2m_0c^2)^{1/2}$. You can see from equation 6.8 that the rate of change of *r* along the optic axis is smaller for more energetic electrons (larger V) and larger for more intense field strengths (larger **B**). Likewise, from equation 6.9, the angular rotation rate increases with increasing field strength and decreases for more energetic electrons.

SPIRAL

The electron spirals through the lens field: a helical trajectory. For electrons with higher keV, we must use stronger lenses (larger **B**) to get similar ray paths.

While these conclusions might be intuitively obvious, the implication is often missed. When we change the TEM accelerating voltage, we change the lenses in the microscope! (Think what this would mean in a VLM.) Therefore, the calibration of the TEM and the lens 'constants' change as we change the kV. Remember the initial paraxial assumption; we'll use non-paraxial rays to explain the effect of spherical aberration on resolution a little later in Section 6.5.A.

While all these ray equations are approximations, they form the basis for more detailed mathematical models of electron motion through lenses (see Chapter 2 in the companion text and URL #2). The more complete models are used in advanced software which simulates the effects of new lens shapes, bore/gap ratios, etc., and has permitted significant advances in the design of lenses to meet the more stringent demands of the latest TEMs.

PITCH OF THE HELIX

When we increase **B**, the pitch of the helical path becomes less if we do not change the energy, because the electrons rotate round the axis more often per unit path length along the axis (*z*).

6.3.D Image Rotation and the Eucentric Plane

So the electrons follow a helical path as they traverse the field along the axis of the lens. This rotation is rarely

shown on standard ray diagrams. You'll see this effect as you operate your TEM because the image or DP rotates on the display screen as you try and focus or if you change magnification. This rotation may require calibration as we'll see in Chapter 9, unless the manufacturer has compensated for it by including an extra lens.

We've already seen in Figure 6.4 that if we change the strength of the lens while keeping d_o fixed, the position of the focal plane and the image plane will also change. Because of this, we have to define a standard object plane for the main imaging lens of the microscope and we call this the *eucentric* plane. Your specimen height should always be adjusted to the eucentric plane because an image of an object in this plane will not move as you tilt the specimen around the primary tilt axis of the holder. (The image will still move if you tilt orthogonally, unless the TEM stage is completely computer controlled to compensate for this.) All other planes in the imaging system are defined with reference to the eucentric plane.

We'll tell you much more about this very important reference plane in Chapter 9.

EUCENTRIC PLANE

If your specimen is in the eucentric plane, then the objective lens strength is always the same when the image on the screen is in focus.

6.3.E Deflecting the Beam

There are many occasions during the operation of the TEM when we want to deflect the beam entering the lens. We may wish to deflect the beam laterally off axis or tilt it to a certain angle with respect to the optic axis. In STEM, these operations are essential to the whole process of forming a scanning image. It is also useful in AEM to be able to blank the beam, i.e., deflect it off axis so it goes into a Faraday cup to measure the current, or to prevent the beam from hitting the specimen when no useful spectroscopic data are being gathered. The way we do this is to apply an electromagnetic field to tilt or traverse the beam or an electrostatic field to blank it. Electromagnetic scan times are of the order of milliseconds while electrostatic blanking can occur in fractions of a microsecond.

Although we are assuming that the lens is thin and has effectively zero thickness along the optic axis, the magnetic field actually acts over a length L. The angle of deflection ε is (for small ε)

$$\varepsilon = \frac{eLB}{mv} \tag{6.10}$$

From this equation we can show that to tilt the beam by 5° we need a coil carrying about 0.2 A and ~100 turns applied along a length of 10 mm, giving a field of 0.01 T. For electrostatic blanking we need about 2 kV/mm.

6.4 APERTURES AND DIAPHRAGMS

We mentioned earlier that an aperture is often inserted into a lens. The aperture limits the collection angle (β) of the lens as shown schematically in Figure 6.10 and such an aperture in the objective lens allows us to control the resolution of the image formed by the lens, the depth of field and the depth of focus, the image contrast, the collection angle of the electron energy-loss spectrometer, the angular resolution of the DP, and so on. In other words, this aperture is important! Physically, the aperture may reside above, in, or below the plane of the lens as we draw it in ray diagrams (but it doesn't really matter since the actual effect will be the same and we've already seen that the electron doesn't care which way it is going). Apertures can also perform other functions, which we'll come across later, such as protecting the specimen from stray radiation in the illumination system, measuring the current in the beam or changing that current.

Usually the apertures are circular holes in metal disks and the disks are made of either Pt or Mo, which are both refractory metals.

A quick word on terminology: While the aperture is the hole in the disk, the metal surrounding the aperture is called the diaphragm (like the variable iris diaphragm in a VLM or your camera). We use the aperture to allow certain electrons to pass through the lens and exclude others by causing them to hit the surrounding diaphragm. This 'aperture/diaphragm' wording, while strictly correct English, is a bit cumbersome, and microscopists tend to be lazy and use 'aperture' in both the correct sense of a hole, but also incorrectly to describe the action of the diaphragm. So we might say that "the objective aperture was used to exclude high-angle scattered electrons from the image" or, as we indicated above, "the aperture protects the specimen from stray radiation" while, strictly speaking, the diaphragm did the excluding and protecting. We'll try to be both consistent and correct in our usage of both terms, but sometimes the precise terminology is awkward.

Diaphragms come in several forms, depending on their function and the particular microscope. They can be either individual disks, each with a particular aperture diameter, or they can be a series of different apertures in a single metal strip (as shown in Figure 6.10). The diameter can be as small as 10 μm, which is about

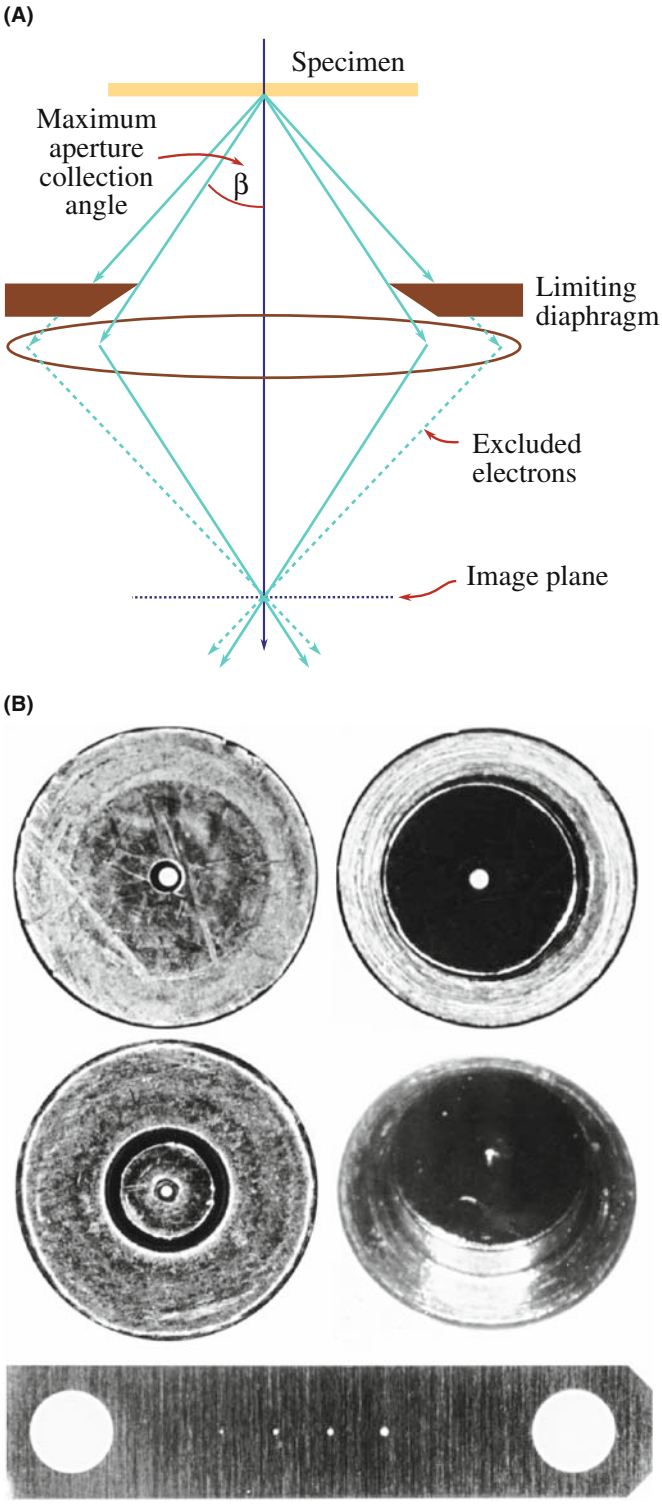

FIGURE 6.10. (A) Ray diagram illustrating how a diaphragm restricts the angular spread of electrons entering the lens. Only electrons emerging from the specimen scattered through angles less than that subtended by the aperture at the object (β) are included in the image-forming process (full lines). The excluded electrons are scattered at angles >β and are stopped by the diaphragm (dotted lines). (B) A selection of diaphragms: the top and middle left are upper and lower views, respectively, of a conventional objective diaphragm; the top/middle right are views of a 'top-hat' (thick) C2 diaphragm; below is a metal strip containing several apertures. Each diaphragm is ~3 mm across.

the smallest circular aperture we can make consistently, or up to ~ 0.3 mm (300 μm). The individual diaphragms or the strips are usually a heavy metal such as Mo or Pt and ~ 25–50 μm thick, but if their job is also to prevent X-rays from hitting the specimen they may be several millimeters thick (see Chapter 33), which means they can be quite expensive if they're made of Pt.

Often the diaphragm collects contamination caused by the electron beam cracking residual hydrocarbons in the vacuum (as we describe in Chapter 8). The contamination tends to accumulate on the edges of the aperture, destroying their circular shape and causing astigmatism. So the diaphragms need occasional cleaning, which can be done by heating them to red heat in the central blue part of a butane flame. In some TEMs, this problem is eliminated by making the diaphragm from very thin metal foil (e.g., Au or Mo). The foil gets hot in the electron beam and any contamination boils off. But such 'self-cleaning' diaphragms are delicate and often crack, thus allowing electrons through other gaps, which defeats the object of the exercise of producing a well-defined aperture.

A SAFETY NOTE ON X-RAYS

X-rays with energies up to the beam energy are generated within the lens wherever the electron beam hits a surface (particularly a limiting diaphragm). So substantial, carefully designed lead shielding is incorporated into the column of the TEM to prevent irradiation of the operator. Obviously, it could be very dangerous to tamper with the lenses or diaphragms of the microscope in any way and only qualified engineers should dismantle, take apart, or repair the lenses or remove the diaphragms.

6.5 REAL LENSES AND THEIR PROBLEMS

It might appear from what we've discussed so far that the analogy between electromagnetic lenses and convex glass lenses is complete, but that is not so. Over the 300 years since van Leeuwenhoek first constructed a light microscope, glass lenses have developed to a point where perfect lenses can be fabricated. In the 80 years since Busch's first magnetic lens, we haven't progressed so far and our lenses are still very imperfect. We've already compared the best electromagnetic lens to the equivalent of using the bottom of a well-known soft-drink bottle as a magnifying glass. Another common description is that if the lenses in your own eyes were as good as our best electromagnetic lens, then you'd be legally blind! So we have to modify all the ideal ray diagrams we've drawn to take into account the imperfections of the lenses. These imperfections all limit the resolution of the microscope but, paradoxically, help us

to get better depth of focus and depth of field from the microscope.

There are many kinds of lens defects (see Chapter 2 in the companion text) and, at one time or another, the effects of all the various defects can be seen in an image or DP. In practice, however, most of us don't need to know about all of them and we'll emphasize the ones that limit the microscope performance in substantial ways. These comprise spherical aberration, chromatic aberration, and astigmatism.

6.5.A Spherical Aberration

The term 'spherical aberration' has almost entered the popular vocabulary since its presence was discerned in the main optics of the Hubble Space Telescope (unfortunately after launch). This defect occurs when the lens field behaves differently for off-axis rays. For our electromagnetic lenses, the further off axis the electron is, the more strongly it is bent back toward the axis. As a result, a point object is imaged as a disk of finite size, which limits our ability to magnify detail because the detail is degraded by the imaging process. As we've told you many times, we can now correct for this aberration, but it still limits the resolution of most TEMs so we need to examine it carefully.

The effects of spherical aberration are shown in Figure 6.11. A point object P is imaged at P′ in the Gaussian image plane. The image is not a point but is instead a central high-intensity region surrounded by a halo of decreasing intensity (similar to Figure 2.11). Spherical aberration is most important in the objective lens because it degrades the detail that we can resolve in TEM images: all the other lenses magnify any error it introduces. It is equally deleterious in the condenser lenses in an AEM or STEM which we use to form the smallest probe with the most current. What we can accomplish at the resolution limits of all forms of TEM is almost always limited by spherical aberration; which is why we're so excited that we can now correct for it.

From Figure 6.11 you can see why we use the term 'spherical' to describe the aberration. The effect of this aberration is to take the curved (spherical) wavefront from the source and increase the curvature. Now if you go back and look at Figure 6.9, you'll see that electrons traveling through a point P on axis intersect the axis again at point P′ where the distance PP′ is given by

$$\mathrm{PP}' = v_2 T_c = v T_c \cos\theta = 2\pi \frac{mv}{eB}$$
$$\left(1 - \frac{\theta^2}{2} + \ldots\right) = L_0\left(1 - \frac{\theta^2}{2} + \ldots\right) \qquad (6.11)$$

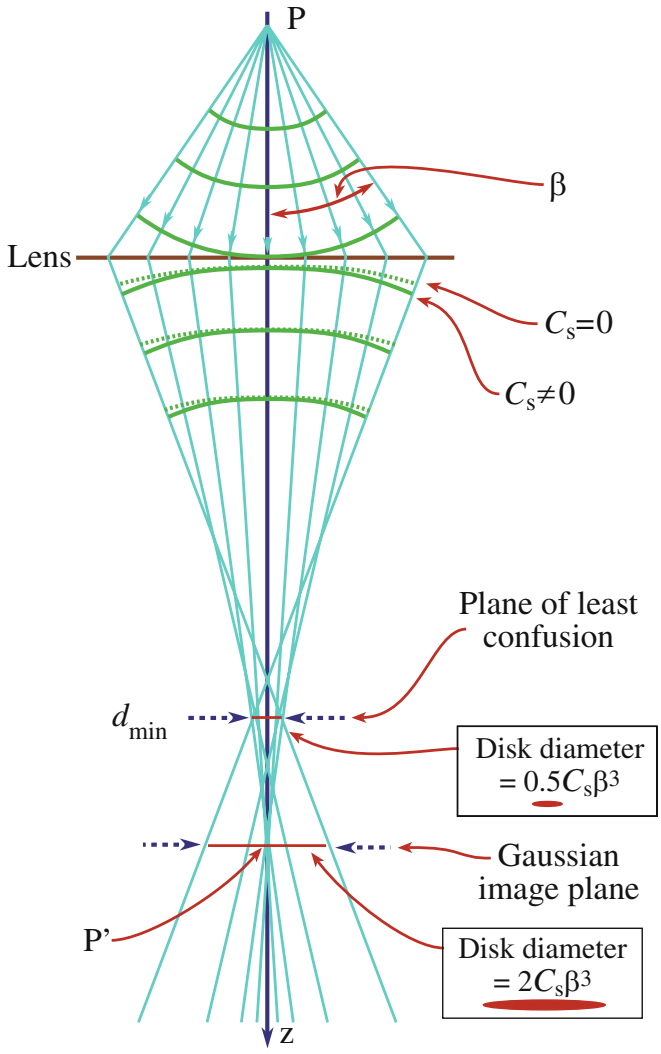

FIGURE 6.11. Spherical aberration in the lens causes wavefronts from a point object P to be spherically distorted by bending the rays at the outside of the lens more than those close to the axis. The point is thus imaged as a disk with a minimum radius in the plane of least confusion and a larger disk at P′ in the Gaussian-image plane. The plane of least confusion is where the smallest image of the object is formed. Schematic intensity distributions at these two important planes are shown beside the ray diagram.

In this relationship, $L_0 = \mathrm{PP}'_0$, where P'_0 is the Gaussian image of the point P for very small θ (i.e., paraxial (near-axis) conditions). As θ increases, the distance PP′ decreases because of spherical aberration and we can write

$$PP' = PP'_0 = -\Delta z \qquad (6.12)$$

where $\Delta z = 0.5 L_0 \theta^2$. Thus we get an expression describing the error, δ, in the Gaussian image position due to spherical aberration

$$\delta = \Delta z \tan\theta \sim \Delta z \theta = 0.5 L_0 \theta^3 \qquad (6.13)$$

So the diameter of the Gaussian image of a point *formed by paraxial rays* is given by this expression, which we will write as

$$\delta = C_s\theta^3 \qquad (6.14)$$

where C_s is a constant (a length) for a particular lens called the spherical aberration coefficient. As you will have noticed, we often use 'C_s' to mean spherical aberration as in 'C_s correction,' 'C_s corrector' and 'C_s corrected.' We'll see in a while that this equation is *very* important because of its effect on the resolution of the TEM and so we need to make a few points of clarification here.

- Equation 6.14 was for paraxial rays only. In a real TEM, the apertures are usually large enough that paraxial conditions do not apply and the sharp image is made more diffuse. As a result the spherically aberrated Gaussian image under non-paraxial conditions is broadened to a diameter of $\delta = 2C_s\theta^3$ (see Figure 6.11).
- You'll often see equation 6.14 written as $\delta = C_s\theta^3 M$ when referring to the image plane but because most discussion of TEM resolution refers back to the minimum distance that we can resolve in the object plane (i.e., the specimen) the magnification term is sometimes left out and we will use this approach.
- When referring to resolution in TEM images it is the *radius* of the point that is more important than the diameter.
- In a real lens the value of θ in equation 6.14 which describes the angle of the electron to the optic axis is replaced by the maximum angle of collection of the (objective-lens) aperture, β.

So, in the forthcoming discussion of resolution, we'll use the radius, we'll refer to the object plane, and we'll use β to define the objective-lens angle of collection. This latter approach is compared to our use of α when discussing beam size in Chapter 5, since α defined the angle of beam convergence. Be careful: many other TEM texts use α somewhat more indiscriminately for both collection and convergence angles.

So we end up with an expression for the radius of the spherically aberrated disk of intensity r_{sph} in the Gaussian image plane, referred back to the specimen plane, under non-paraxial (i.e., realistic) conditions, given by

$$r_{sph} = C_s\beta^3 \qquad (6.15)$$

Because β (in radians) is small, then β^3 is a very strong dependence. The units of r and C_s have to be the same and since C_s is typically a few mm, then we can measure r in (very small fractions of) mm. From this derivation (and linking equations 6.13 and 6.14) you can see that C_s has the dimensions of length and typically it is approximately equal to the focal length of the lens, which for objective lenses in most TEMs is 1–3 mm, but in high-resolution instruments may be well below 1 mm (or not, if they have a C_s corrector).

If you look at Figure 6.11, you will see that the smallest dimension of the cone of rays formed by the lens does not occur at the Gaussian image plane. As we note in the figure, the smallest dimension is formed at a plane closer to the lens which goes by the delightful term 'plane of least confusion' or sometimes 'plane of minimum confusion'; this disk has a radius of $0.25C_s\beta^3$ and a diameter of $0.5C_s\beta^3$. As we'll discuss in Section 6.6.C, some texts use this smaller dimension to define the resolution limit imposed by spherical aberration and it is popular with the TEM manufacturers since it is smaller than the disk in the Gaussian image plane and thus the resolution of the microscope appears better!

CONFUSION

Beware when reading about TEM image resolution because of the confusion between the definition that refers to the Gaussian image plane and that referring to the plane of least confusion.

The way that a corrector compensates for C_s in a magnetic lens is in effect to create a diverging (i.e., concave) lens which spreads out the off-axis beams such that they re-converge to a point rather than a disk in the Gaussian-image plane. In practice this correction is achieved by a highly complex, computer-controlled set of quadrupoles and hexapoles or octupoles. There are two main approaches to the solution of C_s correction. The first is due to the work of Rose and colleagues in Germany embodied in the CEOS commercial system and is used for both probe correction in STEMs and image correction in TEMs. The second is due to Krivanek et al. and is used in Nion dedicated STEMs and has also been retrofitted to several VG STEMs. Figure 6.12 shows schematic ray diagrams for the Nion corrector and the CEOS system. We'll leave a more in-depth discussion of C_s correction to the companion text.

6.5.B Chromatic Aberration

This term is related to the 'color' (i.e., frequency, wavelength, or energy) of the electrons. We've assumed so far that the electrons are monochromatic, but they aren't really. However, we can make very good high-voltage

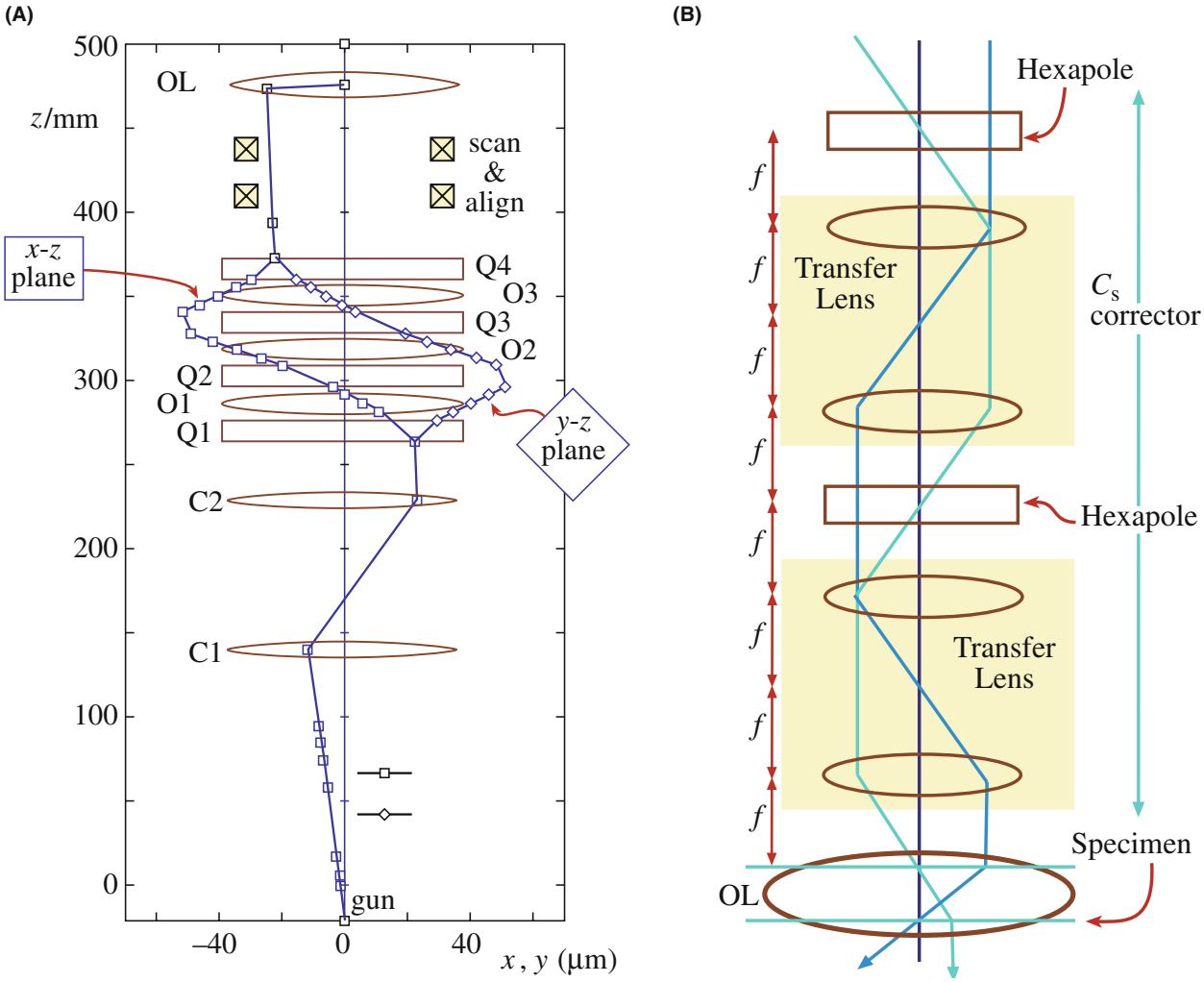

FIGURE 6.12. Ray diagrams showing how the two different commercial systems use (A) multiple quadrupole (Q) and octupole (O) lenses (Nion) or (B) hexapole and other transfer lenses (CEOS) to correct for C_s.

supplies and the variation of the electron energy due to the power supplies is usually smaller than one part in 10^6, which is 0.1 eV for a 100-keV beam. As we discussed in Chapter 5, depending on the electron source the actual energy spread in the beam may vary from ~0.3 eV (cold FEG) to ~1 eV (LaB_6). This range is still so small that we generally don't have to worry about chromatic aberration affecting the image resolution. The exception is if you happen to have a C_s corrector, in which case, after compensating for C_s, C_c is the next most-persistent aberration. Lens elements that can correct for C_c are being developed.

USING A MONOCHROMATOR

Correcting for C_c effects only makes sense if you are dealing with specimens that are thin enough such that specimen-induced chromatic effects do not dominate the resolution. (Correcting C_s is similar.)

Chromatic aberration could be almost completely ignored if we didn't put a specimen into the beam. Unfortunately, this rather essential action creates electrons of a whole range of energies emerging from the thin foil (for reasons we described in Chapter 4). The objective lens bends electrons of lower energy more strongly and thus electrons from a point in the object once again are blurred to form a disk in the Gaussian-image plane (Figure 6.13) (and a smaller disk in the plane of least confusion). The radius r_{chr} of this disk (referring to the object plane) is given by

$$r_{chr} = C_c \frac{\Delta E}{E_0} \beta \tag{6.16}$$

where C_c is the chromatic-aberration coefficient of the lens, ΔE is the energy loss of the electrons, E_0 is the initial beam energy, and β is the angle of collection of the lens. C_c, like C_s, is a length, approximately equal to the focal length. While ΔE in the incident electron beam

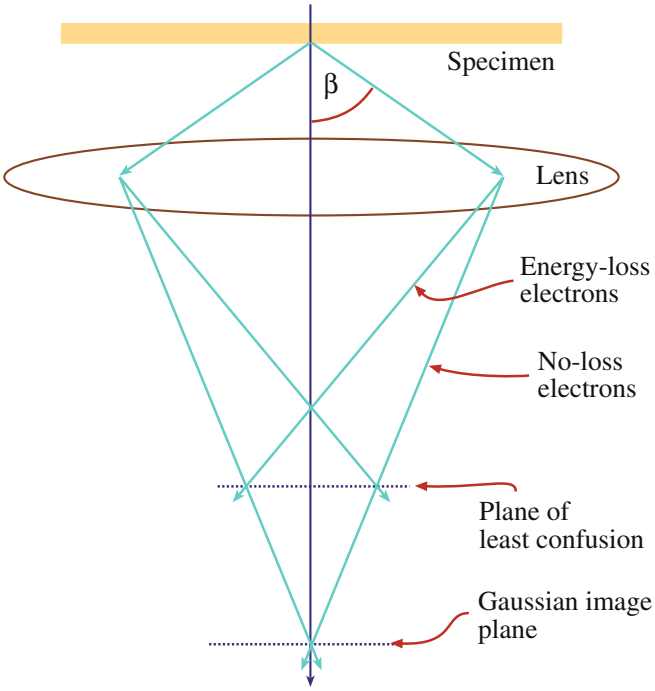

FIGURE 6.13. Chromatic aberration results in electrons with a range of energies being focused in different planes. Electrons emerging from the specimen with no loss of energy are less strongly focused than those that suffered energy loss within the specimen. So, as in Figure 6.11, a point in the object is imaged as a disk in the Gaussian image plane and there is a plane of least confusion.

is $<\sim$1 eV as we just noted, it is typically 15–25 eV for a good fraction of the electrons coming through a typical 50–100 nm thick foil so, as you can easily calculate, r_{chr} is quite a large number (compared to atomic dimensions). Chromatic aberration gets worse for thicker specimens and is better for thinner ones (remember the almost ubiquitous 'thinner-is-better' criterion). So you can do something cheaply to minimize the effects of this aberration; make thin specimens!

The mechanics of C_c correction depend on whether we're trying to compensate for beam effects or specimen-induced effects. We just reminded you that the energy range of electrons coming from the gun is governed by the type of electron source so there are limits depending on which source you have in your TEM. Monochromating the source is a (very expensive) solution. We discuss monochromators at various times in Chapters 37–40 on EELS because that is where such correction pays the greatest dividends.

Unfortunately, for the vast majority of TEM studies, our specimens are not thin enough and, when we have to live with C_c limitations due to having a thick specimen, energy-filtering (EF) is the best solution. EFTEM can correct for the poor resolution that arises when we form images or DPs with electrons that have lost substantial amounts of energy in the specimen, as we'll also discuss in detail in the EELS chapters.

6.5.C Astigmatism

Astigmatism occurs when the electrons sense a non-uniform magnetic field as they spiral round the optic axis. This defect arises because we can't machine the soft-iron polepieces to be perfectly cylindrically symmetrical down the bore. The soft iron may also have microstructural inhomogeneities which cause local variations in the magnetic field strength. Even if these difficulties were overcome, the apertures we introduce into the lens may disturb the field if they are not precisely centered around the axis. Furthermore, if the apertures are not clean, the contamination charges up and deflects the beam. So there are a variety of contributions to astigmatism, which distorts the image by an amount r_{ast} where

$$r_{ast} = \beta \Delta f \tag{6.17}$$

and Δf is the maximum difference in focus induced by the astigmatism. Fortunately, astigmatism is easily corrected using stigmators, which are small octupoles that introduce a compensating field to balance the inhomogeneities causing the astigmatism. There are stigmators in both the illumination (condenser lenses) system and the imaging system (objective lens) and we'll describe how to use them in Chapter 9.

In summary, spherical and chromatic aberration and astigmatism are the three major defects in electromagnetic lenses. There are several minor defects, such as barrel and pincushion distortion, which are self-explanatory in terms of how they distort the image. They are occasionally seen at very low magnification where electrons traveling well off axis and close to the bore of the polepiece appear in the image. Other defects such as coma, and field curvature we will ignore for now.

Again, if you want to learn more about any of these defects they are covered in Chapter 2 in the companion text.

6.6 THE RESOLUTION OF THE ELECTRON LENS (AND ULTIMATELY OF THE TEM)

Another note on terminology: We electron microscopists tend to be rather imprecise in our definition and use of the words 'resolution' and 'resolving power' and related expressions. We've borrowed these terms from classical VLM, which is concerned with the imaging of incoherent light waves through amplitude contrast. High-resolution performance in the TEM is a different matter and traditionally involves phase-contrast imaging of reasonably coherent electron waves, so perhaps we shouldn't be surprised if a different usage has developed. But we should at least define the terms we use. Now in VLM the word *resolution* strictly applies to the act of displaying fine detail in an *image*. The

resolving power of the microscope is the ability to distinguish in the *image* two points, which are closely adjacent in the object. The minimum distance apart of these points in the *object* is the *minimum-resolvable distance*. Since electron microscopists customarily talk about the resolution of the TEM in terms of distances in the *object* (usually a fraction of a nanometer), we should then use the term minimum-resolvable distance but instead everyone says resolution.

Because the lens defects that we've just discussed cause a point object to degrade into a Gaussian image with a finite radius (some combination of r_{sph}, r_{chr}, r_{ast}) they limit the resolution of the electron lens, and hence that of the microscope. The image resolution in the TEM is governed by the ability of the objective lens to image the object, while in the STEM the image resolution is governed by how much beam current we can put into a small probe which is an image of the electron source demagnified onto the specimen. In either case, aberrations limit the resolution.

RESOLUTION AGAIN

We will use the word resolution, but we define it to mean (usually) the minimum-resolvable distance in the object!

6.6.A Theoretical Resolution (Diffraction-Limited Resolution)

If there are *no* aberrations at all, the resolution of *any* lens (glass, electromagnetic, electrostatic. . .) is customarily defined in terms of the Rayleigh criterion, which we introduced back in Equation 1.1 for light optics. Rayleigh's criterion for resolution is arbitrary in the sense that it is not a fundamental physical rule but more a practical definition. This criterion gives us a figure of merit in terms of our eyes' ability to distinguish separate images of two self-luminous, incoherent point sources.

POINTS BECOME DISKS

A single point source will not be imaged as a point, even if no aberrations or astigmatism are present. The finite size of the lens results in diffraction of the rays at the outermost collection angle of the lens, usually defined by the limiting aperture.

This diffraction results in a point being imaged as a disk (called the Airy disk) which has a schematic cross section intensity profile as shown in Figure 6.14A (and also in Figure 2.11). This effect should be familiar to anyone who has encountered basic physical optics. If the two disks overlap so much that they cannot be resolved as in Figure 6.14B, then the points in the object cannot be resolved. Rayleigh assumed that if the maximum from one source lies over the first minimum from the other source, as shown in Figure 6.14C, then your eye can discern this dip as two overlapping images, thus indicating the presence of two separate objects. Under Rayleigh conditions, when the overall intensity profile exhibits a dip in the middle that is above 80% of I_{max}, the two points cannot be resolved. The separation of the two incoherent point sources is then defined as the theoretical resolution of the lens r_{th} and is given by the radius of the Airy disk, which is similar in form to equation 1.1

$$r_{th} = 1.22\frac{\lambda}{\beta} \tag{6.18}$$

BEWARE!

Sometimes in EM texts you'll find the diameter rather than the radius is used. Reason: the beam diameter defines image resolution in STEM and SEM; in TEM, the radius controls the image resolution. (Hence the factor 1.22 in equation 5.10.)

Any standard text on physical optics (which we've already referenced) will show you how to derive this criterion.

Strictly speaking we should not use this equation for electron sources because they are not incoherent. When dealing with high-resolution images, a different approach is used (see Chapter 28). But for our introductory purposes here, we will be content with this approximation.

From equation 6.18 we see that we can get higher resolution if we lower λ or increase β. This terminology can initially be confusing because, as we just did,

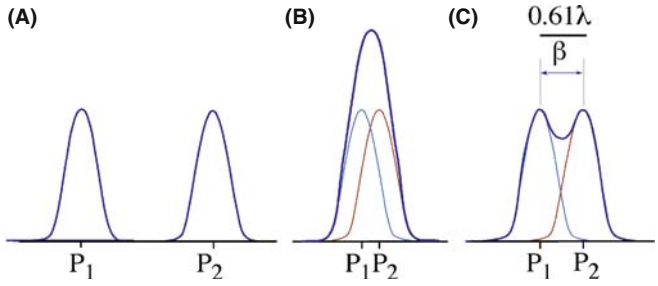

FIGURE 6.14. (A) The Airy-disk intensity profiles from two clearly separated point sources P_1 and P_2. In (B) the two Airy disks are so close that they cannot be distinguished, but in (C) the two are separated such that the maximum in the image of P_1 overlaps the minimum in P_2. This latter situation is the definition of resolution defined by the Rayleigh criterion and is the best (diffraction-limited) resolution.

microscopists often use the expression 'higher resolution' when in fact they mean 'better resolution.' The word higher is then associated with a lower number! It's a smaller r in equation 6.18. It's not just microscopists; a vacuum is also 'higher' if its magnitude is smaller. The improvement in resolution with lower λ is a major reason why there are intermediate and high-voltage TEMs since λ decreases with keV, as we saw back in equation 1.6. The obvious question is, why don't we just increase β (i.e., use a bigger lens aperture or remove it altogether). Well, we could do this if we had perfect lenses, but that isn't the case. All the lens aberrations increase as we increase β (see equations 6.15–6.17); which is why C_s correction is so interesting.

6.6.B The Practical Resolution Due to Spherical Aberration

Let's assume first of all that we have corrected for any astigmatism and our specimen is thin enough that chromatic aberration is negligible. Under these circumstances, the spherical aberration error r_{sph} limits the resolution. Now if you go back and look at equation 6.15 you'll see that r_{sph} increases with the cube of β, a *very* strong dependence. The resolution in the object, then, is given by some combination of the Rayleigh criterion and the aberration error. Hawkes gives a particularly clear description of how this combination leads to a value for the resolution of the microscope. Since this is very often *the* principal figure of merit used when investing hundreds of thousands or even millions of dollars in a TEM, it is essential that you understand that the definition is not exact.

We'll start by summing the radii of the Rayleigh disk and spherical-aberration disk (in the Gaussian image plane) in quadrature (remember it's radii for image resolution, diameters for probe-limited resolutions)

$$r = \left(r_{th}^2 + r_{sph}^2\right)^{1/2} \tag{6.19}$$

Therefore, since both these terms are approximate

$$r(\beta) \approx \left[\left(\frac{\lambda}{\beta}\right)^2 + \left(C_s\beta^3\right)^2\right]^{1/2} \tag{6.20}$$

Since the two terms vary differently with the aperture collection angle β, a compromise value exists when the differential of $r(\beta)$ with respect to β is set to zero and we find that

$$\frac{\lambda^2}{\beta^3} \approx C_s^2\beta^5 \tag{6.21}$$

So we come up with an optimum expression for β which Hawkes (1972) gives as

$$\beta_{opt} = 0.77\frac{\lambda^{1/4}}{C_s^{1/4}} \tag{6.22}$$

The exact value of the numerical factor depends on the assumptions made about the various terms included in the definition of resolution and so is often written simply as ***A***. Sometimes, this compromise value is determined by simply equating the equations for r_{th} and r_{sph} rather than going through the summation in quadrature. A quick calculation for 100-keV electrons (λ = 0.0037 nm) for an instrument with C_s = 3 mm gives a β_{opt} value of ~4.5 mrads.

If this expression for β_{opt} in equation 6.22 is substituted into equation 6.20 we get a minimum value of $r(\beta)$

$$r_{min} \approx 0.91\left(C_s\lambda^3\right)^{1/4} \tag{6.23}$$

This is the expression we want; it gives the *practical* resolution of the TEM.

The numerical factor in equation 6.23 is often written as ***B***. Typically, the value for r_{min} is ~0.25–0.3 nm and the best high-resolution instruments have r_{min} ~0.1–15 nm; 1-Å TEMs are about the best available without C_s correction and about 0.07 nm is (at the time of writing) the best reported resolution with C_s correction. So, as we showed back in Figure 1.2, we can resolve rows of atoms, which in most crystalline materials have a separation close to r_{min} (although low-index planes in some metals are still below this resolution). It's worth noting that since your eyes can resolve a distance of ~0.2 mm, then the maximum useful magnification of the best high-resolution TEM is $\sim 3\times10^6$. Above this magnification, no more detail will be revealed.

Hawkes (1972) reminds us that the decision to add in quadrature back in equation 6.19 was arbitrary, and simply summing r_{th} and r_{sph} is another possible way to determine r_{min} (as we'll see in Section 28.7). But any way you combine the two terms for r (or diameter if you're discussing a probe-limited resolution) leads to expressions that have the same general form as equation 6.22 for β_{opt} and equation 6.23 for r_{min}. In some cases, ***A*** and ***B*** are put equal to unity and not even included, and if you're not pushing any limits in your calculations or experiments this latter approach is a very reasonable approximation.

As we indicated right at the beginning of our discussion, electron microscopists are rather imprecise in our definition of the resolution. However, the resolution is often given as a very precise number!

6.6.C Specimen-Limited Resolution Due to Chromatic Aberration

Remember that we assumed in the previous section that there was no contribution from chromatic aberration. However, if you have a thick specimen then a significant number of electrons will lose 15–25 eV of energy (a typical value of the most probable (plasmon) energy loss; see Figure 4.1). If you put 20 eV into the chromatic-aberration resolution limit given by equation 6.16 you'll find that, at 100 keV with β_{opt} of 4.5 mrads from equation 6.16, the value of r_{chr} is ~2.5 nm.

C_C-LIMITED RESOLUTION

This is typically 10× larger than the C_s-limited resolution. When you're looking through a thick specimen the performance of your TEM is 10× worse than its specified resolution.

If you have a thick specimen, it doesn't matter what voltage you use or how low your C_s is; it doesn't matter if you've got a 1-MeV TEM or access to a C_s corrector; you'll have an image resolution in the 1–3 nm range and you can see all the available information in your specimen at a magnification as little as $\sim 10^5\times$. In fact, the vast majority of all recorded TEM images have C_c-limited resolution: your images will too!

So how thick is thick? Well, it depends on TEM voltage and the mean free path for elastic and inelastic scatter in your specimen, which increases with Z (see Chapter 4). For good high resolution at 100 keV your specimen should be $<\sim$ 30 nm, while at 300 keV you can probably get away with ~50 nm before C_c effects begin to control resolution, assuming $Z <\sim 30$. So for higher-Z specimens the 'thinner is better' axiom is even more important. A more restrictive rule of thumb given by Sawyer and Grubb is that, for biological and polymeric specimens, the resolution limit is about one tenth the specimen thickness. As we noted when we first talked about chromatic aberration, the solution to this problem is in your hands (and in Chapter 10).

6.6.D Confusion in the Definitions of Resolution

If you're new to the subject, you don't have to read this section because it may confuse you still further, but if you've read other TEM texts you may have noticed discrepancies in the definitions of resolution.

We used the expression for r_{sph}, measured at the Gaussian-image plane. Strictly speaking, it is only under ideal conditions (i.e., if $C_s = 0$) that we should use the Gaussian image as a measure of the resolution limited by the lens and it is only really correct to use the Gaussian image under *paraxial* conditions, that is with a *very* small objective aperture. As we've already noted, in the TEM β is usually large enough that paraxial conditions do not apply. So the disk in the plane of least confusion is the relevant feature from which to define the best image resolution, as shown back in Figure 6.11.

If this is so, why did we choose the definition of r_{sph} as the radius of the disk in the Gaussian image plane?

The answer to this question is discussed by Hawkes. Defocusing the image slightly, to bring the plane of least confusion to the Gaussian image plane, will indeed lead to a decrease in the value of the numerical factor in equation 6.23 from 0.91 to 0.43. Hawkes also comments that since this latter value is smaller, manufacturers tend to use it to define the resolution of their instrument! However, this whole treatment of resolution assumes incoherent illumination, which is *not* the case in the TEM. Also, the resolution depends on the contrast in the image and how the lens transfers information from the object to the image. As a result, Hawkes concludes that (see equation 6.23) $\boldsymbol{B} \sim 1$ (from the Gaussian image) is "a more prudent choice" (i.e., closer to reality) than $\boldsymbol{B} = 0.43$ (from the disk in the plane of least confusion) even though, strictly speaking, the plane of least confusion refers to the conditions operating in the TEM.

So it is basically a matter of opinion whether to use the diameter or the radius of the disk in the Gaussian image plane or that in the plane of least confusion. Fortunately, it doesn't really matter too much since, in the end, the choice only alters the value of the numerical terms $\boldsymbol{A}$ and $\boldsymbol{B}$, which we've already mentioned are often approximated to unity anyhow. For example, the value of $\boldsymbol{A}$ will depend on exactly which of the several quoted expressions was used for r_{sph}, e.g., if there was 0.25, 0.5, or 1 in front of $C_s\beta^3$. After these various terms are fed into the equations and the value of β_{opt} is extracted, $\boldsymbol{A}$ only varies by about ±15%. A small variation in $\boldsymbol{B}$ will occur also, for the same reason.

BEWARE!

1. There are inconsistencies in the definition of the terms used to describe the effects of C_s on TEM resolution
2. We use the Gaussian image radius referred back at the object plane, i.e., we use $r_{sph} = C_s\beta^3$.

We have tried to be consistent in our use of the radius of the Airy disk and the radius of the aberration/astigmatism error. Obviously, it doesn't really

matter whether you use the radius or the diameter, so long as you are consistent. Occasionally, however, you may find the Airy disk *radius* is used in combination with the *diameter* of the disk in the plane of least confusion or the Gaussian image plane, so this also contributes much to the discrepancy between various TEM texts.

The question to any student learning HRTEM is: do you know what your resolution really is?

6.7 DEPTH OF FOCUS AND DEPTH OF FIELD

You should have got the message that, because of the poor lens quality we have to use small apertures to minimize their aberrations. This generally means that we cut out many of the electrons that would otherwise be gathered by the lens. Fortunately, our electron sources are so bright that we can live with substantially reduced beam currents hitting our specimen. In fact there are advantages to using small apertures, despite the price we pay in image intensity, probe current, and diffraction-limited resolution. These advantages come in the form of better depth of focus and better depth of field. These terms can be confusing, and once again, the TEM literature is variable. So we need to go back to physical optics to find the correct definition of these terms.

Basically, we are trying to find out how much of the object (the specimen) is in focus at the same time and over what range the image is in focus. (This latter question is irrelevant in SEM and dedicated STEMs without post-specimen lenses where we don't use conventional lenses to form the image, so both terms are equivalent.) In TEM both terms are important.

The depth of field, D_{ob}, is measured at, and refers to, the *object*. It's the distance along the axis on both sides of the object plane within which we can move the object without detectable loss of focus in the image. The depth of focus, D_{im}, is measured in, and refers to, the *image plane*. It is the distance along the axis on both sides of the image plane within which the image appears focused (assuming the object plane and objective lens are fixed). Note that we say "appears" in both cases and this of course also depends on how good your eyes are.

We can derive expressions for these definitions using Figure 6.15. Imagine that ray 1 originates at the highest point up the column where the object can appear to be in focus within the resolution and that this ray arrives at the farthest point down the column where the image can appear to be in focus. Ray 2 represents the other extreme but travels at the same inclination to the optic axis. If these two rays appear to come from the same point (to within the resolution of the lens) the distances d_{ob} and d_{im} correspond to the smallest distances which we can resolve in the object or image, respectively. Note immediately that d_{im} is greater than d_{ob}. Now we can show that angles α_{im} and β_{ob}, which are both small, are given by

$$\alpha_{im} \approx \tan \alpha_{im} = \frac{d_{im}/2}{D_{im}/2} \tag{6.24}$$

and

$$\beta_{ob} \approx \tan \beta_{ob} = \frac{d_{ob}/2}{D_{ob}/2} \tag{6.25}$$

The angular magnification is thus

$$M_A = \frac{\alpha_{im}}{\beta_{ob}} \tag{6.26}$$

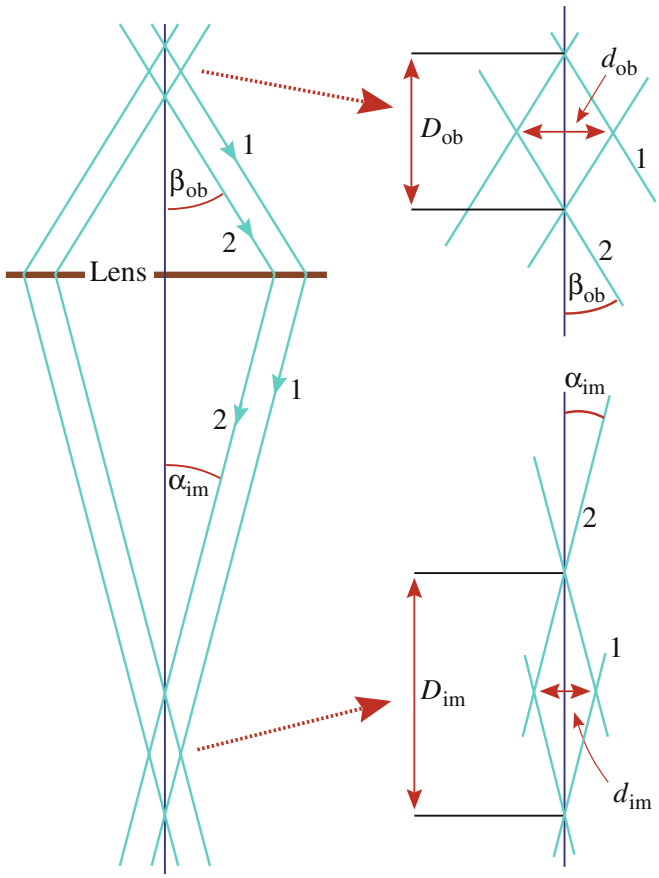

FIGURE 6.15. The definition of the depth of field and the depth of focus. Rays 1 and 2 represent the extremes of the ray paths that remain in focus when emerging $\pm D_{ob}/2$ either side of a plane in the specimen. Typically D_{ob} is greater than the specimen thickness. The same rays define the depth of field over which the image is in focus $\pm D_{im}/2$ either side of the image plane. The resolution in the object is d_{ob} and that in the image is d_{im}.

and the transverse magnification (which we simply call the magnification) is

$$M_T = \frac{d_{im}}{d_{ob}} \tag{6.27}$$

If these two magnifications are related in the usual way by

$$M_T = \frac{1}{M_A} \tag{6.28}$$

Then we can say that the depth of focus is given by

$$D_{im} = \frac{d_{ob}}{\beta_{ob}} M_T^2 \tag{6.29]}$$

and the depth of field is

$$D_{ob} = \frac{d_{ob}}{\beta_{ob}} \tag{6.30}$$

Notice that for a correct calculation of either D_{ob} or D_{im} you must be careful to select the right value of β. Under different circumstances, the limiting angle is defined by the illumination aperture α (in the C2 lens) or the objective aperture β_o (in the objective lens). In thin specimens, which scatter weakly, most electrons emerge from the specimen in a cone closer to that defined by α_{im}, which is often very small ($\sim 10^{-4}$ rad). In a thicker, more strongly scattering specimen, the objective aperture defines the angle and β_o is usually about 10^{-2} rad.

FOCUS AND FIELD

So we get a much greater depth of focus and field by using small apertures (small β).

For a collection angle, β_{ob}, of 10 mrad and a d_{ob} of 0.2 nm, equation 6.30 tells us that the depth of field will be 20 nm, i.e., a specimen of this thickness can all be in focus at the same time. If you only need 2-nm detail in your image, then you can use a specimen which is 200 nm thick and it will still all be in focus.

If we want to see detail at the 0.2 nm level we need to use a magnification of about 500,000×. Equation 6.29 tells us that, under these conditions, the depth of focus will then be 5 km! If we only need to see 2 nm, we can use a magnification of 50,000× and the depth of focus is 5 m. In either case, we have tremendous latitude in where we put the photographic negative or CCD camera because it would still record a focused image many meters either side of the screen. This explains why we can use a CCD camera which can be inserted just below the final projector lens, and still get a focused image with a TV camera well below the standard film camera. In fact, the TEM image would be in focus on the floor under the microscope (or maybe even the floor below if your TEM lab has the misfortune not to be on the lowest floor) if you projected it there but M_T would be different!

Now things get a little more complicated if you're using a C_s-corrected TEM because, for example, in STEM we can use much larger condenser apertures which give much larger convergence angles and thus define the probe a lot more strictly, over distances less than the specimen thickness. In this case, rather than worrying about the reduced depth of field it becomes feasible to think about focusing the probe at different levels within the specimen to explore any structural or chemical variations through the foil thickness. So now we can think about overcoming the projection limitation of TEM images. (Remember the rhino?) The first attempts at imaging single atoms at specific depths within a thin specimen have already been reported.

CHAPTER SUMMARY

We've introduced you to the principles of how an electromagnetic lens works, and how we describe its functions in simple ray diagrams. There are two principal operations: either we use the lens to create an image of an object or we use it bring parallel rays to a focus. We'll see in later chapters that the former operation is used to create magnified images of the specimen on the screen of the TEM. It is also used to create small electron probes (demagnified images of the electron source) at the plane of the specimen in a STEM or AEM. The latter operation is used to create DPs in the back-focal plane of the objective lens.

Our lenses are rather abysmal in their performance, resulting in the need for small limiting apertures. The lens aberrations limit the resolution of the microscope and we usually need an optimum aperture to get the minimum resolution. The small apertures cut down the electron beam intensity, but also give us remarkable depth of focus and depth of

field in our images and specimen, respectively. The recent development of aberration correctors for TEM will revolutionize much of what we've written in this chapter. However, very few TEMs are equipped with correctors and so, for the vast majority of users, it is important to understand the limits imposed on the resolution performance of TEMs by their lenses and by your specimens.

You don't need much skill to use a magnifying glass. The manufacturers may say the same for today's TEM. We say, the more you know about the TEM, the more you'll get out of it and the less likelihood you'll have of making embarrassing errors.

SOME HISTORY

Busch, H 1927 *Über die Wirkungsweise der Konzentrierungsspule beider braunschen Röhre* Arch. Elektrotech. **18** 583–594. The original paper on focusing electron beams.

Hawkes, PW (Ed.) 1997 *Advances in Imaging & Electron Physics Vol. 100: Partial Cumulative Index* Academic Press New York (now published by Elsevier). Essential reference for the historically minded.

Hawkes, PW 2004 *Recent Advances in Electron Optics and Electron Microscopy* Ann. Fond. Louis de Broglie **29** 837–855. An overarching yet concise review of recent advances in electron optics and microscopy, with a great collection of references, both historical and recent.

LENSES AND ELECTRON TRAJECTORIES

Reimer gives a summary of lens defects and more on the derivation of equation 6.11.

Grivet, P 1972 *Electron Optics* Pergamon Press New York.

Hawkes, PW 1972 *Electron Optics and Electron Microscopy* Taylor & Francis Ltd. London. This account is particularly clear if you have an interest in the physics of electron lenses. An important discussion of how to take account of many aberrations when giving a figure of merit. In 'Confusion in the Definitions of Resolution,' we follow Hawkes' clear reasoning regarding the plane of least confusion.

Hawkes, PW (Ed.) 1982 *Magnetic Electron Lenses* Springer New York. A collection of review articles in true Peter Hawkes style; thorough, sound, erudite, and informative.

Hawkes, PW and Kasper, E 1989, 1994 *Principles of Electron Optics* **1**–**3** Academic Press New York. Comprehensive but advanced. Volume **3** includes imaging in the TEM. If by now you're getting the idea that Hawkes is *the* source of electron optical information, then you are right.

Klemperer, O and Barnett, ME 1971 *Electron Optics* Cambridge University Press New York.

Munro, E 1997 *Electron and Ion Optical Design Software for Integrated Circuit Manufacturing Equipment* J. Vac. Sci. Technol. B **15** 2692–2701. More on electrons moving through the lens.

Rempfer, GF 1993 *Electrostatic Electron Optics in the 1940s and Today* MSA Bull. **23** 153–158. By an expert in the use of electrostatic lenses.

ABERRATION CORRECTION

The companion text goes into this in much more detail. In particular, you'll find there that C_s is actually better written as C_3. There are many more 'C_s' terms. These references give an introduction.

Chang, LY, Kirkland, AI and Titchmarsh, JM 2006 *On the Importance of Fifth-Order Spherical Aberration for a Fully Corrected Electron Microscope* Ultramicroscopy **106** 301–306.

Krivanek, OL, Delby, N and Lupini, AR 1999 *Towards Sub-Å Electron Beams* Ultramicroscopy **78** 1–11. Used in the Nioen STEM.

Urban, K, Kabius, B, Haider, M and Rose, H 1999 *A Way to Higher Resolution: Spherical-Aberration Correction in a 200 kV Transmission Electron Microscope* J. Electr. Microsc. **48** 821–826.

RESOLUTION

All texts on TEM will include a discussion of resolution. Particularly useful are those in Reimer 1997, Edington 1976, Fultz and Howe 2002 and Hirsch et al. 1977.

❑ Points to be wary of when reading about definitions of C_s-limited resolution: (see references in Chapter 1)

Sawyer and Grubb (2008) and Egerton 2005 use the Gaussian image radius referred back at the object plane, just as we do; i.e., $r_{sph} = C_s\beta^3$. Reimer 1997 and Fultz and Howe 2001 use the diameter of the disk in the plane of least confusion; i.e., $d_{sph} = 0.5C_s\beta^3$ although both also describe the radius at the Gaussian image plane as we do. Beware: Edington 1976 implies, and Hirsch et al. 1977 state, that $C_s\beta^3$ is the radius *of the disk in the plane of least confusion, which it is not, since by definition it must be less than the Gaussian-image radius (see Figure* 6.11).

Sawyer, LC, Grubb, DT and Meyers, DT 2008 *Polymer Microscopy* 3rd Ed. Springer New York. Rule of thumb for polymers.

❑ Points to be wary of when reading about depth of field and depth of focus

Bradbury et al. 1989 give a particularly clear discussion of the topic. Reimer 1997 uses the term depth of focus for the depth of field and uses depth of image for depth of focus; a rare inconsistency! The terms are used interchangeably in SEM because there is no lens between the object and the image.

Bradbury, S, Evennett, PJ, Haselmann, H and Piller, H 1989 *Dictionary of Light Microscopy* Royal Microscopical Society Handbook #15 Oxford University Press New York. For the VLM comparison.

SPECIAL TECHNIQUES

Borisevich, AY, Lupini, AR, Travaglini, S and Pennycook, SJ 2006 *Depth Sectioning of Aligned Crystals with the Aberration-Corrected Scanning Transmission Electron Microscope* J. Electr. Microsc. **55** 7–12. Moving to confocal imaging in the TEM.

THE COMPANION TEXT

The concepts of depth of field and depth of focus are explored in greater depth in Chapter 2 of the companion text. There, you'll also find more on the principles of lens optics and on Newton's lens equation in particular. Lenses may also be used in other components of the TEM such as electron spectrometers or, in effect, the electron gun; you'll find more on these topics in the same chapter. Quadrupoles, sextupoles, and octupoles are critical components in the correction of aberrations and in some spectrometers; we decided to leave even an introduction to the details until the specialized chapters in the companion text.

URLs

1) http://www.opticsinfobase.org/default.cfm. Optics information base courtesy of the Optical Society of America; lists of papers and journals.
2) http://www.mebs.co.uk/about_us.htm. Munro's company site; provides commercial software for electron optics; essential for the serious designer.

SELF-ASSESSMENT QUESTIONS

Q6.1 How do you focus an image in a TEM?
Q6.2 What kind of visible-light lens does the behavior of a magnetic lens resemble?
Q6.3 Name the main components of a magnetic lens and state their functions.
Q6.4 What are the back and front-focal planes of a magnetic lens?
Q6.5 What do we mean by the term 'optic axis'?
Q6.6 What force acts on an electron in a magnetic field and how can we control this force?
Q6.7 What effect does the magnetic lens have on the trajectory of the electron with respect to the optic axis?
Q6.8 To achieve the highest magnification, where should the specimen be located relative to the objective lens?
Q6.9 Define 'underfocused' and 'overfocused.'
Q6.10 Why is the objective lens the most important lens in a TEM?
Q6.11 Define the eucentric plane.
Q6.12 Explain the difference between a diaphragm and an aperture.
Q6.13 Why do we use apertures in the TEM?
Q6.14 What causes spherical aberration and how can we minimize it?
Q6.15 Define chromatic aberration and describe how to minimize it.
Q6.16 What causes astigmatism and how do we correct it?
Q6.17 Define resolution (strictly speaking, the resolving power) of the TEM?
Q6.18 What ultimately limits the TEM resolution?
Q6.19 In practice, what often limits the practical TEM resolution?
Q6.20 In the TEM, what is depth of field, what controls it, and why is it important?
Q6.21 In the TEM, what is depth of focus, what controls it, and why is it important?

TEXT-SPECIFIC QUESTIONS

T6.1 Estimate the limit of resolution in a 100-kV TEM if the specimen is very thin. Assume $C_s = 1$ mm and $\beta = 10$ mrads.
T6.2 Under the same conditions, estimate the limit of resolution if the specimen is thick enough so that each electron on average undergoes a plasmon loss of ~ 15 eV.
T6.3 If your specimen is pure Al how thick does it have to be such that each electron typically suffers a single plasmon loss. (Hint: go back to Chapter 4.)
T6.4 Go on the Web to find the image resolution offered by commercial manufacturers for a typical 200-kV TEM. (Find Web pages for FEI, Hitachi, JEOL, Zeiss.) Compare the resolution with your answers to questions 1

and 2. What does this exercise tell you about the assumptions that are being made when a TEM resolution is specified?

T6.5 Use suitable values of β_{ob} and α_{im} to deduce values for D_{ob} and D_{im} under the following two conditions: (a) 100-keV electrons, 20 k× magnification, looking for 1 nm detail in the image; (b) 200-keV electrons, 800 k× magnification, looking for 0.2 nm detail in the image.

T6.6 Examine Figure 6.9. Why is B exactly parallel to the optic axis? Is the electron traveling exactly parallel to B? Show, with diagrams, that you understand the reason for these questions.

T6.7 In deriving equation 6.14 we say this is used only for paraxial rays. Why?

T6.8 Explain why the electrons in Figure 6.13 are 'overfocused' if they have lost energy.

T6.9 Discuss the accuracy of using a Faraday cup to measuring beam current.

T6.10 Why do we always draw electron lenses as convex and why haven't we been able to build a concave lens for electrons using cylindrically symmetric lenses? (Hint: recently this problem has, in effect, been solved and has resulted in a solution to the long-standing problem of reducing spherical aberration.)

T6.11 Just for the heck of it try to draw Figure 6.1 to scale assuming that the focal length and object distance are about 3 and 1.5 mm, respectively, and the aperture in the bore of the lens is 60 μm diameter. Estimate values of α and β and explain which of the various lens properties wouldn't be such a good choice for a real lens in TEM.

T6.12 Calculate the radius of the spiral trajectory of 100- and 300-kV electrons in a magnetic field of 1 T.

T6.13 Use ray diagrams to distinguish the terms underfocus and overfocus. Usually it is good to operate any kind of microscope with the lenses in focus. Can you think of any occasion when underfocus (or overfocus) conditions might be useful? (Hint: refer to Figures 6.4 and 6.5.)

T6.14 Why do we use soft magnets and not permanent magnets for electron lenses? If we were in fact to use a permanent magnet what advantages might this bring to the design of a TEM?

T6.15 Why do we have to cool the electron lenses? List as many drawbacks as you can to having to cool the lenses. Can you think of any lenses that might not require cooling? Could we design a TEM in such a way that lens cooling is not required?

T6.16 How could you compensate for the rotation introduced into an electron beam by the action of the lens field? Why would you want to do this?

T6.17 Distinguish the plane of least (or minimum) confusion, the Gaussian image plane, the back-focal plane and the front-focal plane of a lens, using diagrams where necessary. (Hint: Figure 6.11 is a good place to start.)

T6.18 Explain (using diagrams where necessary) why paraxial rays from a point in an object are not subject to significant spherical aberration, yet are still brought to a focus in the Gaussian image plane as a disk rather than as a point.

T6.19 Calculate the radius of the image disk in the plane of least confusion and in the Gaussian image plane under spherical aberration conditions. Assume reasonable values for all terms and justify your assumptions.

T6.20 Calculate the optimum semi-angle of collection for the objective aperture to minimize the contributions of spherical aberration in a 200-keV microscope. State any assumptions. What is the practical resolution of the TEM under these conditions?

T6.21 How is the practical resolution of a TEM further compromised if thick specimens are used? Calculate the expected resolution (i.e., the radius of the disk in the plane of least confusion in the Gaussian image plane) assuming all the electrons suffer an energy loss of $\sim$ 15 eV. Assume reasonable values for all terms and justify your assumptions.

T6.22 Why are the terms *depth of field* and *depth of focus* distinctly differently in TEM, but used interchangeably in SEM?